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ON THE ORIGINS OF RUSSELL'S THEORY OF DESCRIPTIONS

BY ANDREW PETER REBERA

THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF
PHILOSOPHY (DPHIL)

UNIVERSITY OF SUSSEX

MAY 2010

I hereby declare that this thesis has not been, and will not be, submitted in whole or in part to another university for the award of any other degree.

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SUMMARY

This thesis explores the development of Bertrand Russell's theory of definite descriptions. It aims at demonstrating the connection between Russell's views on the subject of denoting and his attempt, in the period 1903-05, to develop a solution to 'the Contradiction' (i.e. the Russell Paradox). The thesis argues that the discovery of the theory of descriptions, and the way in which it works, are best understood against the backdrop of Russell's work on the paradoxes. A new understanding of Russell's seminal paper 'On Denoting' is presented, including a novel interpretation of the notorious 'Gray's *Elegy* Argument', in which Russell argues against his earlier theory of denoting.

That Russell's work on denoting is connected to his work on the paradoxes is reasonably well-known: the nature of the connection has not, however, been adequately brought out in the literature. This is addressed through demonstrating the relationship between Russell's work on denoting and his development of the 'substitutional theory' of classes and relations. This theory eliminates classes and propositional functions in favour of matrices and substitutions. The role of the theory of descriptions in the development of the substitutional theory is commonly supposed to be merely that the theory of descriptions facilitates the ontological elimination of classes. But this elimination was equally possible on Russell's earlier theory of denoting (which he had rejected in the Gray's *Elegy* Argument). In the thesis it is suggested that the theory of descriptions brings with it a new conception of analysis, and that it is through the introduction of this new form of analysis—rather than through the elimination of classes—that the theory of descriptions facilitates the substitutional approach.

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List of Abbreviations for Russell's Works

In the thesis, Russell's works are referred to by the following abbreviations. Full details may be found in the bibliography.

<i>Auto</i>	<i>Autobiography</i> (Russell 2000)
<i>Classes</i>	'Classes' (Russell 1903e)
<i>CP3</i>	<i>The Collected Papers of Bertrand Russell</i> , vol. 3 (Russell 1993b)
<i>CP4</i>	<i>The Collected Papers of Bertrand Russell</i> , vol. 4 (Russell 1994a)
<i>Difficulties</i>	'On Some Difficulties in the Theory of Transfinite Numbers and Order Types' (Russell 1906a)
<i>DVD</i>	'Dependent Variables and Denotation' (Russell 1903b)
<i>EIP</i>	'The Existential Import of Propositions' (Russell 1905a)
<i>FN</i>	'Fundamental Notions' (Russell 1904b)
<i>Functions</i>	'Functions' (Russell 1903f)
<i>IMP</i>	<i>Introduction to Mathematical Philosophy</i> (Russell 1993a)
<i>Insolubilia</i>	'On "Insolubilia" and their Solution by Symbolic Logic' (Russell 1906c)
<i>KAKD</i>	'Knowledge by Acquaintance and Knowledge by Description' (Russell 1910-1911)
<i>ML</i>	'Mathematical Logic as Based on the Theory of Types' (Russell 1908)
<i>MMD</i>	'My Mental Development' (Russell 1951)
<i>MPD</i>	<i>My Philosophical Development</i> (Russell 1959)
<i>MSOR</i>	'Mr Strawson on Referring' (Russell 1957)
<i>MTCA</i>	'Meinong's Theory of Complexes and Assumptions' (Russell 1904a)
<i>OD</i>	'On Denoting' (Russell 1905c)
<i>OF</i>	'On Fundamentals' (Russell 1905b)
<i>OMD</i>	'On Meaning and Denotation' (Russell 1903d)
<i>OMDP</i>	'On the Meaning and Denotation of Phrases' (1903a)
<i>OnF</i>	'On Functions' (Russell 1904c)
<i>PAD</i>	'Points about Denoting' (Russell 1903c)
<i>PLA</i>	'The Philosophy of Logical Atomism' (Russell 1918)
<i>PM</i>	<i>Principia Mathematica</i> (Whitehead & Russell 1927)

<i>PoL</i>	<i>A Critical Exposition of the Philosophy of Leibniz</i> (Russell 1958)
<i>PoM</i>	<i>The Principles of Mathematics</i> (Russell 1937)
<i>PoP</i>	<i>The Problems of Philosophy</i> (Russell 2001)
<i>STCR</i>	‘On the Substitutional Theory of Classes and Relations’ (Russell 1906b)
<i>TK</i>	<i>Theory of Knowledge: The 1913 Manuscript</i> (Russell 1992)

Introduction

This thesis concerns the origins of Bertrand Russell's 'theory of descriptions'. Its main aim is to demonstrate the connection between Russell's views on denoting and his attempt, in the period 1903-05, to develop a solution to 'the Contradiction' (i.e. the Russell paradox). In this way the thesis will, I hope, make a significant contribution to the understanding of the theory of descriptions and of the classic paper 'On Denoting' (*OD*) in which it was first presented.

Two connections are established in the thesis. The first is between Russell's earlier and later theories of denoting: between the 'theory of denoting concepts' from 1903's *The Principles of Mathematics* (*PoM*), and the theory of descriptions from *OD*. The second is between the theory of denoting (generally conceived) and the attempt to solve the paradoxes. I will speak to each of these connections, and then provide an overview of each of the five chapters that follow.

1. Denoting

In the broadest sense, *denoting* is a certain kind of phenomenon: that phenomenon whereby description is possible¹; that phenomenon whereby a sentence may express a proposition which is about an entity in virtue of that entity's satisfying a certain property. Thus 'denoting phrases' are phrases formed by the combination of a determiner expression ('some', 'all', 'any', 'the', etc.) with a noun phrase; and the occurrence of a denoting phrase in a sentence signals that the proposition it expresses is, in some sense, about some selection of entities having the property given by noun phrase. It is in this sense that *OD* is a paper about denoting, and in this sense that the theory of descriptions is a theory of denoting.

The notion of denoting is introduced in *PoM* as a fundamental logical relation, one of the constants to be appealed to in Russell's attempt to demonstrate the logicist thesis that the truths of mathematics can be deduced from pure logic. This technical sense of denoting is to be distinguished from the broad sense identified above. In the technical sense denoting is a relation which holds between a 'denoting concept' and an entity just

¹ Cf. *PoM*: §56, 53.

in case, when the denoting concept occurs in a proposition, the proposition is about the entity. This is the basic thought behind *PoM*'s theory of denoting concepts.

The theory of descriptions and the theory of denoting concepts are both *theories of denoting*. I will use the expression 'theory of denoting' in this broad sense; thus if I, for example, speak of the connection between *the theory of denoting* and the paradoxes, I should be understood as making a claim concerning the relation of the paradoxes to *the general theory of the phenomenon of denoting*. If I intend a particular theory of denoting I will refer to it by name.

The notion of denoting is intimately linked to the notion of *generality*. A theory of denoting will constitute an account of generality, or at least of propositions involving generality. Recognising this is crucial to understanding the relationship between *OD* and Russell's earlier work. In *PoM* all denoting phrases—definite descriptions included—are taken to indicate denoting concepts and so to involve generality. In this they are to be contrasted with singular terms. Now on a fairly commonplace reading of *OD*, the purpose of the paper is to question the status of definite descriptions as singular terms.² The referential use of definite descriptions is, after all, very common; and this observation has formed the basis of most major objections to Russell's theory.³ But if Russell's view prior to *OD* was that the analysis of sentences containing definite descriptions involved the notion of generality, it begins to seem less likely that the arguments in *OD* explicitly concern singular terms. This suspicion can only be reinforced by the discovery—now widely accepted—that the 'Gray's *Elegy* Argument (GEA)⁴, *OD*'s main objection to rival views, primarily targets Russell's earlier position.⁵ I take it, therefore, that in order to approach a full understanding *OD*, one must appreciate both the theory of denoting concepts and the argument which led Russell to abandon it.

² E.g. Salmon 2005.

³ Most obviously Strawson 1950 and Donnellan 1966.

⁴ *OD*: 48-51.

⁵ That this is the case is obvious from the manuscript 'On Fundamentals' (*OF*), written in 1905 prior to *OD*.

2. Denoting and the Paradoxes

In *My Philosophical Development* (MPD), Russell described the period during which he arrived at the theory of descriptions as follows:

When *The Principles of Mathematics* was finished, I settled down to a resolute attempt to find a solution of the paradoxes. [. . .] Throughout 1903 and 1904, my work was almost wholly devoted to this matter, but without any vestige of success. My first success was the theory of descriptions, in the spring of 1905 (MPD: 79)

Whilst much has been written about the theory of descriptions, comparatively little has been written about its origins. Recent interest in the GEA has done much to redress the balance, and attention has refocused on Russell's earlier account of denoting. However this does not explain why he should have taken the theory of descriptions to be his 'first success' in the attempt to find a solution of the paradoxes. Why, if his work was 'almost wholly devoted' to the attempt to find a solution of the paradoxes, was Russell investigating the nature of denoting and of descriptions? On this very central question, the literature is, on the whole, silent.⁶

But perhaps this silence is to be explained by the simple fact that there is no immediate connection between the paradoxes and denoting at all. Immediately after the passage quoted above, Russell says that the theory of descriptions 'was, apparently, not connected with the contradictions, but in time an unsuspected connection emerged' (MPD: 79). Perhaps it was just good old luck that brought two apparently unconnected areas of Russell's work together.

But this suggestion does not bear close scrutiny. Why was Russell working on denoting at all, if it bore no relation to his overall project of solving the paradoxes? His work on denoting was clearly no passing fancy, as the posthumously published papers from the period 1903-05 attest.⁷ Moreover, the paradoxes, in their most well-known formulations, involve classes and predicates: is it, then, just a coincidence that in *PoM* the discussions of denoting, classes, and predicates are so closely linked?⁸

⁶ By no means *completely* silent. See, e.g., Rodriguez-Consuegra (1989; 1992); Wahl (1993); Landini (1998a; 1998b); and Levine (2005).

⁷ Cf. *OMDP*, *DVD*, *PAD*, *OMD*, *OF*; see also *EIP*.

⁸ I discuss this connection in Chapter One.

I think it is plain that the paradoxes and the theory of denoting *are* connected. The question is not so much *if* they are connected, but *how*. The answer concerns Russell's 'substitutional theory of classes and relations' (*STCR*). Russell had tried to construct a substitutional theory in 1904, but had, he claimed, 'failed for want of the theory of denoting' (Grattan-Guinness 1977: 79-80). Once the theory of descriptions was up and running, however, he 'found at last that substitution would work, and all went swimmingly' (Grattan-Guinness 1977: 80). The key to understanding the relation between the theory of denoting and the paradoxes is, I think, to understand why it is that the substitutional theory would only work with the new theory of denoting, and not with the old. If we could account for this we would, it seems to me, have identified what, for Russell, was most important about the theory of descriptions.

3. Overview of the Following Chapters

Chapter One: Denoting in the Principles of Mathematics

Russell's general metaphysical and logical framework is set out. It is argued that the theory of denoting concepts is introduced as a basic component of Russell's logical apparatus—as part of an account of *generality*—rather than in response to an epistemological problem concerning infinite classes (as has been suggested in the literature). The 'logical genesis view' of the theory of denoting concepts is introduced and defended; in so doing, the close connection between denoting concepts, predicates, and classes is emphasised. Russell's interest in denoting is located in a firmly logico-mathematical setting.

Chapter Two: Denoting and the Paradoxes

It is argued that Russell's presentation of the 'mixed' paradox of classes (involving classes-as-many and classes-as-one) involves a complex interplay of the notions of intension and extension. (This interplay is related back to the theory of denoting in Chapter Five.) The standards that Russell demanded of a satisfactory solution to the paradoxes are examined. In illustrating Russell's concerns in this area, a widespread understanding of his commitment to the doctrine of the unrestricted variable is challenged. The 'theory of meaning and denotation' is introduced and shown to differ very little from the theory of denoting concepts. The role of the theory of meaning and denotation in the search for a solution of the paradoxes is then discussed.

Chapter Three: 'On Denoting' and the Theory of Descriptions

The theory of descriptions is introduced, and its central principle taken to be that *denoting phrases are incomplete symbols*. It is argued that the notion of an incomplete symbol, at least as Russell employs it, is intimately bound to the notion of 'structurally-radical interpretive analysis'. The neglect of this connection has, it is argued, induced a tendency to mischaracterise the nature of Russell's theory. Emphasising the connection serves to distance the theory *as Russell presents and uses it*, from the version advocated by contemporary philosophers of language. It is suggested that Russell's version of the theory—by treating predicate expressions as what I shall call 'quasi-incomplete symbols'—is suitable for his purposes apropos the paradoxes in a way that the more contemporary version is not (this is discussed again in Chapter Five). The theory of descriptions' solution of *OD*'s three 'logical puzzles' is then demonstrated; and the 'Central Question of *OD*' is identified as being: How is it that sentences containing denoting phrases come to be about whatever it is that they are about?

Chapter Four: The Gray's Elegy Argument

A detailed original interpretation of the 'Gray's *Elegy* Argument' (GEA) is provided. It is argued that the argument targets both Russell's earlier theory of denoting and Frege's theory of *Sinn*. These two theories share a common assumption. This assumption is not, however, the thesis that definite descriptions are singular terms (as argued by Nathan Salmon (2005)); rather, both theories appeal to the relation between meaning and denotation in their respective answers to the Central Question of *OD*. Each paragraph of the GEA is then discussed, and the argumentative moves uncovered. The argument is taken to concern the ability of the theory of meaning and denotation to adequately answer the Central Question of *OD*. It is suggested that the GEA is sufficient to warrant the rejection of Russell's earlier view, but that Frege's theory can, if one is so inclined, be rescued.

Chapter Five: Toward a Solution to the Paradoxes

The thematic connections between the theory of denoting, the GEA, and the paradoxes are established. Certain structural similarities between Russell's thinking about classes and denoting are indicated, and some connections are drawn between the interplay of the notions of extension and intension in relation to the theory of classes and the theory

of denoting. The role of the theory of descriptions in relation to the ‘substitutional theory’ is then discussed. Here it may be seen why it is important to distinguish between the theory of descriptions as presented by Russell, and the modified theory endorsed by contemporary philosophers of language. For it is the element of structurally-radical interpretive analysis—i.e. that which contemporary philosophers of language fail to carry over from Russell—that is the key to understanding Russell’s use of the theory of descriptions (rather than the theory of meaning and denotation) in his substitutional response to the paradoxes.

1. Denoting in *The Principles of Mathematics*

1. Introduction

Prior to the discovery of the theory of descriptions, Russell endorsed a theory of denoting he had first set out in his 1903 work *The Principles of Mathematics* (*PoM*). The discovery of the theory of descriptions was intimately connected with the discovery of certain ‘rather curious difficulties’ (*OD*: 48) in the earlier view, difficulties concerning its ability to adequately discharge its duties as a theory of denoting.

So what are the duties of a theory of denoting? What is a theory of denoting supposed to do? This chapter will set about answering these rather general questions by introducing *PoM*’s ‘theory of denoting concepts’, and addressing the question of why Russell introduced it.

One answer to this question is that endorsed by James Levine in the following passage:

Russell broke with idealism toward the end of 1898, but he introduced the theory of denoting concepts sometime in 1901, only after coming to accept Cantor’s theory of the infinite. *The introduction of denoting concepts was a response to certain epistemological difficulties raised by this change in Russell’s early philosophy.* (Levine 1998: 416, emphasis added)

There is certainly some merit in this view, for Russell did hold that the theory of denoting concepts resolved the epistemological worries to which Levine refers.¹ However I shall be proposing an alternative account of the origins of the theory of denoting concepts (and so of Russell’s interest in denoting). According to the view I shall defend, the notion of *denoting* is central to Russell’s account of *generality*. The theory of denoting concepts is introduced not in response to any specific epistemological problem, but as a basic component of his logical apparatus. That there is some connection between *denoting* and *generality* is acknowledged by some commentators.² But the precise nature of the connection has not been discussed in the

¹ *PoM*: §72.

² Hylton 2005d: 195-96; Wahl 2007: 12; Landini 2009: 285.

literature. I will describe how Russell ‘derives’ (in a certain sense of that word) denoting concepts from predicates, and will stress the connection between denoting concepts, predicates, and classes. The notions of *denoting* and of *class* are, for the Russell of *PoM*, basic logical notions to be used in the logicist reduction of the truths of mathematics to truths of logic. They share a deep connection in virtue of a common link to the nature of predicates.³

The present chapter has two main aims. The more general aim is to introduce the basic Russellian framework within which this entire thesis is situated. To this end I begin, in §§2-3, by introducing Russell’s general metaphysical and epistemological position in *PoM*. The second, more particular, aim is to investigate the nature of Russell’s interest in denoting. In §3 I discuss the epistemological problem to which the above quotation from Levine refers. In giving a brief primary account of the theory of denoting concepts, I explain how it addresses this problem. In §4 I argue for what I shall call ‘the logical genesis view’ of the origins of the theory of denoting concepts. The view favoured by Levine and others—the ‘epistemological view’ as I shall call it—rests on an equivocation. I shall argue that in identifying and resolving this equivocation, one renders the epistemological view poorly motivated.

2. Terms, Propositions, and Acquaintance

Having broken free from neo-Hegelian idealism, Russell and G. E. Moore embraced an extreme realism.⁴ On this view the world is ultimately composed of a number of discrete, self-subsistent, mind- and language-independent entities, which Russell calls ‘terms’.

Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual and entity. The first two emphasize the fact that every term is *one*, while the third is derived from the fact that every term has being, *i.e. is* in some sense. A man, a moment, a number, a class, a relation, a chimaera,

³ They also share a deep connection to propositional functions. Although I say very little about propositional functions in this chapter, they feature frequently in later chapters.

⁴ Russell often acknowledged his intellectual debt to Moore: e.g. ‘On fundamental questions of philosophy, my position, in all its chief features, is derived from Mr G. E. Moore’ (*PoM*: xviii) (cf. *MTCA*: 21; *MPD*: 54). The precise extent to which Moore led the way is a matter of dispute (Baldwin 1990: 6-7; Cartwright 2003: 109; Griffin 1991: §7.2).

or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false. (*PoM*: §47, 43)

Terms are possible objects of thought but, as Moore is careful to point out, this is not a defining feature of them, but ‘merely states that they may come into relation with a thinker’ (Moore 1899: 179).⁵

Terms are the constituents of propositions, and propositions—complexes of terms, unified in a distinctive manner—are the objects of judgement and meanings of declarative sentences.⁶ The crucial difference between terms and propositions is that the latter are capable of truth or falsehood, while the former are capable of neither. The proposition that *Jones is a madman* is (say) true, while the simple term *Jones* is not the kind of thing that could have a truth-value. For Russell, during this period, truth is not a matter of correspondence between a proposition and something else. The proposition is composed of the very entities to which the words in the sentence refer⁷; hence there is nothing left over for the proposition to correspond to. Rather, Russell’s view is that truth is an un-analysable property of some propositions:

It may be said – and this is, I believe, the correct view – that there is no problem at all in truth and falsehood; that some propositions are true and some false, just as some roses are red and some white. [. . .] What is truth, and what falsehood, we must merely apprehend, for both seem incapable of analysis. (*MTCA*: 75-6)

If this account of truth is to be at all convincing, Russell owes an explanation of the distinction between simple terms and propositions.⁸ Moore had attempted to explain the distinction between simple terms and propositions by way of relations:

⁵ He adds that ‘in order that they *may* do anything, they must already *be* something’ (Moore 1899: 179), emphasising their independence of the mind. Moore makes his comments regarding what he calls ‘concepts’. Moore’s concepts are not precisely the same as Russell’s terms, but, for our immediate purposes here, the differences are unimportant.

⁶ Not all complexes of terms are propositions (e.g. classes-as-many are non-propositional complex objects); but all *unified* complexes of terms are propositional (*PoM*: §439, 466). Note that Russell follows Moore in holding that the object of perception is an existential proposition (*MTCA*: 21; cf. Moore 1899: 183).

⁷ Thus Russell insists, as against Frege, that Mont Blanc—the actual mountain, snowfields and all—is a component part of the proposition expressed by the sentence ‘Mont Blanc is more than 4000 metres high’ (Frege 1980: 169).

⁸ Russell’s account of truth is difficult in a number of respects. For insightful discussion see Korhonen 2009.

A proposition is constituted by any number of [terms], together with a specific relation between them; and according to the nature of this relation the proposition may be either true or false (Moore 1899: 180).

On this view, that propositions are truth-apt is attributable to their complexity; and the distinction between complex and simple terms is that the former, but not the latter, contain relations. Such a view leads straight to the *problem of the unity of the proposition*, of which Russell later commented: ‘I recognize that it is my duty to answer [the problem] if I can, and, if I cannot, to look for an answer as long as I live’ (Griffin 1993: 159 (Russell to Bradley, 30 Jan 1914)). Russell encountered the problem on many occasions throughout his career⁹, but here is one statement of it¹⁰:

Consider [. . .] the proposition “*A* differs from *B*.” The constituents of this proposition, if we analyze it, appear to be only *A*, difference, *B*. Yet these constituents, thus placed side by side, do not reconstitute the proposition. The difference which occurs in the proposition actually relates *A* and *B*, whereas the difference after analysis is a notion which has no connection with *A* and *B*. [. . .] A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition. (*PoM*: §54, 49-50)

Moore’s explanation of unity does not bear close scrutiny. On his view, if proposition *p* is composed of terms x_1, \dots, x_n , there is some relation *r*, also occurring in *p*, such that *r* relates x_1, \dots, x_n one to another. The nature of *r* will determine *p*’s truth-value, but of *r*’s nature nothing more can be said (Moore 1899: 180). This isn’t really an *explanation* at all. Moreover, the idea that propositional unity can be explained in this way succumbs to Bradley’s Regress. If *p* is composed of x_1, \dots, x_n and relation *r*, then *r* is the explanation of the relatedness of x_1, \dots, x_n . But what is the explanation of the relatedness of x_1, \dots, x_n and *r*? We may, if we like, posit a further relation, *r*’, such that *r*’ relates x_1, \dots, x_n and *r*, but this only leads to the demand for a further relation, *r*’’, and then *r*’’, and so on.¹¹

Like Moore, Russell is convinced that propositional unity is derived from the nature of the relation: unlike Moore, he attempts to clarify its status. Relations, he suggests,

⁹ Hylton 2005b; Griffin 1993; Stevens 2005.

¹⁰ Cf. *PoM*: §§55, 136, 439; *MTCA*: 28; *MPD*: 63.

¹¹ Cf. Bradley 1930: Ch. II.

can occur in propositions in a different manner to other terms; and it is in this difference of mode of occurrence that the explanation of propositional unity is to be found. But before we discuss distinctions among terms and their modes of occurrence in propositions, it will be helpful to turn first to Russell's account of the relation between mind and world.

2.1. *Acquaintance*

Terms are radically independent of the mind, which is to say that they are what they are independently of their being thought of. The mind is *in* the world—it is a term among other terms—but it is not, in any significant sense, *constitutive of* the world. This sets the realism of Russell and Moore clearly apart from the idealism against which they were reacting, by drawing a distinct line between the mind and its objects. But if this separation is maintained *and* genuine knowledge of reality is to be possible, there must be some cognitive capacity capable of spanning the gap between mind and world. That cognitive capacity is *acquaintance*.¹²

From Russell's perspective, what is crucial about *acquaintance* is that it is a direct, unmediated relation between a mind and a term, the converse of the relation of *presentation* between a term and a mind (*KAKD*: 201). Thus to have acquaintance with Socrates is to be (or have been) directly aware of Socrates, for him to have been presented to one. The possibility of this unmediated cognitive relation between mind and term underwrites the possibility of genuine knowledge of terms.¹³ But to have acquaintance with Socrates is not, in itself, to know any fact about him, but only to stand in a direct cognitive relation to him. To know of a term *t* that it has property *F*, is to know a proposition, $\langle t \text{ is } F \rangle$, whose constituents include *t* and *F*.¹⁴ Acquaintance is 'prior' to judgement in the logical sense that it is presupposed by it: one must have acquaintance with *t* in order to entertain a judgement about it. Russell notes that while

¹² Russell says very little about epistemological matters in *PoM*. Explicitly epistemological discussions begin to appear more frequently in the *CP4* manuscripts (esp. *PAD* and *OMD*).

¹³ The contrast with Frege's position serves to bring out the sense in which acquaintance is, for Russell, *unmediated*. On Frege's view, to grasp a thought about Mont Blanc is to grasp a Thought in which occurs a *Sinn* (Sense) which stands in a certain relation to Mont Blanc. Thought is in this way mediated by *Sinne*. For Russell there is no such intermediary: a thought about Mont Blanc involves a proposition in which the mountain itself occurs. Thought must be unmediated in this sense, Russell claims, for 'if we do not admit this, then we get the conclusion that we know nothing at all about Mont Blanc' (Frege 1980: 169).

¹⁴ I use angle brackets (' \langle ' and ' \rangle ') to distinguish propositions (worldly items) from sentences (linguistic items).

acquaintance is logically independent of judgement, ‘it would be rash to assume that human beings ever, in fact, have acquaintance with things without at the same time knowing some truth about them’ (*PoP*: 25).

Propositions, though complex, are terms nonetheless, and one may have acquaintance with them. Having acquaintance with a proposition *p* is not, however, equivalent to judging that *p* for, as we have said, to have acquaintance with a term is not yet to know anything *about* it (while to judge that *p* is to take a stand on its truth-value). Acquaintance is a form of ‘knowledge of things’, while judgement is a form of ‘knowledge of truths’¹⁵ (*PoP*: chs. 4-5 *passim*).

Acquaintance is an unmediated cognitive relation between a mind and a term; its linguistic counterpart is *reference* or *indication*. *Indication* is a direct, unmediated semantic relation between a word and its meaning—and the meaning of a word is a term: ‘*Words* all have meaning, in the simple sense that they are symbols which stand for something other than themselves’ (*PoM*: §51, 47). The meaning of a (declarative) sentence is a kind of concatenation or complex of the meanings of the words in the sentence: that is, the meaning of a sentence is a proposition. The sentence ‘Brutus murdered Caesar’ contains three words ‘Brutus’, ‘murdered’, and ‘Caesar’, which indicate the three terms Brutus, *murder*, and Caesar respectively. The sentence expresses the proposition ⟨Brutus murdered Caesar⟩, a proposition containing (let us suppose) just Brutus, *murder*, and Caesar.¹⁶ But there is more to a proposition than just its constituents.

For one thing, there is, as we have seen, something that explains why those constituents constitute a proposition rather than a mere juxtaposition of terms (though we have not yet seen what, for Russell, this is). But there is also an issue concerning the *structure* of the proposition. The propositions ⟨Caesar murdered Brutus⟩ and ⟨Brutus murdered Caesar⟩ contain precisely the same constituents, but are obviously distinct. The distinction is a matter of their respective structures. This is seen most clearly by

¹⁵ Of course not all judgements constitute knowledge.

¹⁶ The parenthetical caveat alludes to the fact that the proposition may also contain some instant (or instants) of time, corresponding to the tense of the verb. Maybe there are other constituents corresponding to other grammatical properties of the sentence, but for simplicity let us follow Russell’s lead in ignoring such factors.

returning to the sentential level: if one wishes to express the proposition \langle Brutus murdered Caesar \rangle , one needs a sentence containing the words ‘Brutus’, ‘murdered’, and ‘Caesar’; but it is absolutely essential that they are structured appropriately (the verb in the middle, ‘Brutus’ to its left, ‘Caesar’ to its right). Propositions are not just unified complexes of terms: they are structured, such that if two distinct propositions p and q have the same constituents, they may be distinguished by way of their structural properties.

To judge a proposition is not simply to have acquaintance with it, nor yet simply to have acquaintance with all of its constituent terms, but is to have acquaintance with all of its constituent terms *and* some insight into their structure. The capacity for insight into propositional structure is presumably un-analysable (at least there is nothing in *PoM* to suggest otherwise); it amounts to the capacity to recognise, for any proposition that one can entertain, the conditions under which that proposition would be true. That is to say, there is a relation in which one may stand to a proposition whereby one need not have made a judgement as to its truth-value, but whereby one knows what would have to be the case in order that it be true. Call that relation ‘entertainment’: then if S entertains proposition p , S knows p ’s truth-condition. If S judges that p , then S takes p ’s truth-value to be *Truth*. That S judges that p entails that S entertains p ; and both S ’s judging that p and S ’s entertaining p entail that S has acquaintance with all of the constituents of p . These are the cognitive relations between minds, terms, and propositions that the realism espoused by Russell (and Moore) must posit in order to maintain the possibility of genuine knowledge of a radically mind- and language-independent world.

2.2. Analysis and propositional structure

In *The Philosophy of Leibniz* (*PoL*), published in 1900, Russell had written: ‘That all sound philosophy should begin with an analysis of propositions, is a truth too evident, perhaps, to demand a proof’ (*PoL*: §7, 8). In *PoM* analysis is primarily taken to involve the *decomposition* of a complex entity into the simple (or simpler) entities of which it is

composed. The goal of this kind of analysis is to reveal the fundamental constituents of propositions, and hence of the world.¹⁷

An air of paradox surrounds this notion of analysis, for the terms yielded by the analysis of a proposition do not, placed side by side, reconstitute the proposition. Analysis does not, therefore, bring to light the precise nature of the proposition: it yields the constituents of a proposition, without explaining the distinctive unity in which, prior to analysis, they were bound. Moreover, since a proposition is ‘essentially a unity’ (*PoM*: §54, 50) analysis is inherently falsifying, in the sense that it transforms a proposition into that which, by its very nature, it is not (i.e. a *unity* of terms becomes an *aggregate* of terms; that which is inherently *one* becomes that which is inherently *many*). Russell is aware of this but takes it as indicating only that analysis has its limitations, not that it should be rejected as a valid philosophical method. Analysis, he writes, ‘gives us the truth, and nothing but the truth, yet it can never give us the whole truth’ (*PoM*: §138, 141).

Before we come to the analysis of any particular proposition, a word or two about how analysis should be carried out. Russell’s concern is directed explicitly towards propositions as opposed to sentences: towards the world itself rather than any representation of it. But from a practical point of view, analysis is almost unavoidably the analysis of a sentence. One may use a pencil to write a sentence expressing a certain proposition upon a page, but one cannot *put* a proposition *on* a page or *in* a book. The assumption is therefore that sentences can represent propositions, and that the manipulation of a sentence by an analyst may represent the manipulation of the proposition that sentence represents. The view is that there are sufficient similarities of structure and constituency between a sentence and the proposition it expresses that the analysis of the proposition can be represented by the analysis of the sentence. I quote Russell’s account of *philosophical grammar* at length.

The study of grammar, in my opinion, is capable of throwing far more light on philosophical questions than is commonly supposed by philosophers. Although a grammatical distinction cannot be uncritically assumed to correspond to a

¹⁷ This kind of analysis assumes different forms depending upon one’s ontological sensibilities. For Russell the world is composed of terms, and analysis reveals these. For Frege, the world is composed of functions and objects: his version of compositional analysis reveals these.

genuine philosophical difference, yet the one is *prima facie* evidence of the other, and may often be most usefully employed as a source of discovery. Moreover, it must be admitted, I think, that every word occurring in a sentence must have *some* meaning: a perfectly meaningless sound could not be employed in the more or less fixed way in which language employs words. The correctness of our philosophical analysis of a proposition may therefore be usefully checked by the exercise of assigning the meaning of each word in the sentence expressing the proposition. On the whole, grammar seems to me to bring us much nearer to a correct logic than the current opinions of philosophers; and in what follows, grammar, though not our master, will yet be taken as our guide. (*PoM*: §46, 42)

In acknowledging that grammar is only the guide and not the master, Russell acknowledges that there is a higher authority against which the correctness of an analysis is to be gauged. In a footnote to the above passage, he suggests that ‘The excellence of grammar as a guide is proportional to the paucity of inflexions’ (*PoM*: §46, 42n.). This suggests that inflexions are, at least potentially, misleading; but to be aware that this is so is to have the capacity to notice the points at which grammar fails as a guide. This latter capacity presupposes an independent insight on the part of the analyst into the structural properties of the proposition in question. But then grammar can reveal nothing to us that we did not already know (or at least have access to); it may act as a guide, but only on a route for which we already have a reliable map, if only we trouble ourselves to consult it.

The philosophical grammar of *PoM* looks, to our eyes, extraordinarily naive. Russell seems somehow to have slipped from the acceptable claim that:

- (i) the analysis of a proposition *p* can be represented by a series of operations upon (manipulations of) a sentence *s* (where *s* expresses *p*);

to the claim that:

- (ii) the analysis of a sentence *s* can (in general) be taken as a reliable guide to the analysis of *p* (where *s* expresses *p*).

Claim (i) is a basic assumption common to all philosophy in the analytical tradition.

Claim (ii), understood as a claim about sentences of English, is not a basic

assumption—and we have Russell himself to thank for pointing this out in *OD* (this was his great contribution, according to Wittgenstein (1922: 4.0031)). But although Russell soon came to drop the assumption of claim (ii) (understood as a claim about sentences of English), what he never abandoned was the claim that a sentence has *something* in common with the proposition it expresses:

syntax—i.e. the structure of sentences—must have some relation to the structure of facts, at any rate in those aspects of syntax which are unavoidable and not peculiar to this or that language. (*MPD*: 157)

It is this thought that underwrites (i): we can represent the analysis of a proposition by using a sentence of English, but often only by using English in such a way that it is forced to mirror the structural properties of the proposition, leading to unwieldy and unnatural modes of expression. Alternatively we can employ a formal language whose syntax has been developed expressly in order that propositional analysis may be accurately represented by the manipulation of its sentences. In employing such a language—assuming always that it has been accurately developed—the propositions themselves can be left behind, so to speak.¹⁸ As such, (ii), restricted to such a formal language, may be reinstated. But of course (ii) is reinstated only on the understanding that the syntax of the formal language has been developed on the basis of a prior understanding of the structural, logical properties of the propositions one intends to analyse. There is therefore a very clear sense in which, for Russell, all sound philosophy should begin with an analysis of *propositions*' (*PoL*: §7, 8, emphasis added)—it cannot begin with an analysis of sentences unless those sentences are sentences of a suitably developed language, or, if not, have been manhandled into a suitable form.

There is a good deal more to be said about Russell's conceptions of analysis, philosophical grammar, and the idea of a language suitable for philosophy.¹⁹ For now the above must suffice. I shall discuss these matters in a little more detail in Chapter Three, comparing Russell's position in *PoM* with his later view in *OD*.

¹⁸ Cf. definition: 'as soon as the definition is found, it becomes wholly unnecessary to the reasoning to remember the actual object defined, since only concepts are relevant to our deductions' (*PoM*: §63, 63).

¹⁹ For an excellent discussion of these matters see Hylton 2007.

2.3. *Things and concepts: occurrence as subject, occurrence as concept*

Now that the basics of Russell's *PoM* account of the relation between sentences and propositions has been set out, let us return slowly to the question of propositional unity. Russell accepts from Moore the idea that the unity of the proposition is to be explained by way of a relation; but he goes further than Moore in distinguishing between different ways in which a term can occur in a proposition. Ultimately Russell will suggest that propositional unity is explained by the fact that in every proposition at least one term occurs in such a way as to unite the other constituent terms. Whether this constitutes any real progress remains to be seen. Beforehand, we must bring out Russell's distinction between the different modes of occurrence that certain terms may have in a proposition.

Every term is the logical subject of some proposition (for instance, every term *t* is the logical subject of a true proposition of the form *(t is a term)*). As I shall use the expression, the *logical subject* of a proposition is *always* a constituent of that proposition. We shall see presently that denoting concepts introduce complications; for, where denoting concepts are concerned, it may be the case that a proposition is about a term that is not a constituent of it. In such cases I will *not* describe the term that the proposition is about as its logical subject. (Nothing hangs on this stipulation, but it is as well to be clear.) When a term occurs in a proposition as its logical subject²⁰, the proposition may be thought of as asserting some property of that term—of *being* the assertion that such and such is the case. Here Russell has in mind a *logical* sense of assertion, distinct from the more familiar *psychological* or *linguistic* senses. In these senses, thinkers and language users assert propositions whenever they make statements, or believe, doubt, or worry that such-and-such is the case. In Russell's logical sense however, a proposition is described as 'asserted' if it stands in a certain sort of internal relation to its truth-value: in Russell's words it 'in some way or other contains its own truth or falsehood as an element' (*PoM*: §52, 48). This is not a very clear explanation; indeed a clear explanation is not to be found in *PoM*, and is difficult to reconstruct. The thought, roughly, is that propositions, when they are not embedded in other propositions, present themselves as being the case. We might say that they occur in (a

²⁰ Or as one of its logical subjects—propositions may have more than one logical subject.

non-linguistic analogue of) the indicative mood. Thus proposition ⟨1⟩ is said to assert that Socrates is mortal, and hence one judging that ⟨1⟩ judges that Socrates is mortal.

⟨1⟩ ⟨Socrates is mortal⟩

⟨1⟩ presents the mortality of Socrates as actually being the case—it presents itself as being true (whether or not it is true). However, what ⟨1⟩ asserts to be the case—*the mortality of Socrates*—can occur in other propositions in ‘unasserted’ form. For example, the sentence ‘If Socrates is mortal, then someone will perish’ expresses a proposition asserting that *the mortality of Socrates* implies that someone will perish. Here *the mortality of Socrates* has more of the character of an assumption than an assertion.²¹ One judging that ⟨if Socrates is mortal, then someone will perish⟩ does not judge that Socrates is mortal. Russell will describe *the mortality of Socrates* as a ‘propositional concept’ (sometimes as an ‘unasserted’ proposition). The propositional concept *the mortality of Socrates* is not distinct from proposition ⟨1⟩: it is proposition ⟨1⟩ occurring—to return to the analogy employed above—in a different (i.e. non-indicative) mood. Propositional concepts bear a merely external relation to the truth-values to which, *qua* asserted propositions, they bear an internal relation.

Propositions may be analysed into subject term(s) and that which is asserted of them. Unhelpfully, Russell proceeds to call the part of the proposition which is asserted (in the sense of ‘predicated’) of the subject term(s) the ‘assertion’. He then claims that the term which ensures that the proposition as a whole is an asserted proposition (rather than an unasserted propositional concept) will be found among the part of the proposition that he now calls the ‘assertion’. Thus in ⟨1⟩, the subject term is Socrates, and the assertion is the rest of the proposition, less Socrates (i.e. *is mortal*). The term which ensures that the proposition as a whole is an asserted proposition is then to be found among *is mortal*. The assertion *is mortal* is composed of two constituents (corresponding to the two words in its linguistic expression). Thus although a

²¹ Russell’s unasserted propositions bears obvious comparison with Meinong’s ‘assumptions’ (*Annahmen*), which he (Russell) discusses in *MTCA*.

proposition may be analysed into subject and assertion, this will not be ultimate, since the assertion will, in general, be further analysable.²²

Russell identifies three parts of speech as of particular importance in discussions of logic and the nature of the proposition: substantives, adjectives, and verbs. In line with his philosophical grammar, these grammatical distinctions correspond to logical distinctions among terms. Substantives such as proper names correspond to a class of terms called *things*. The distinguishing feature of things is that they can only occur in a proposition as the subject of an assertion, never as the assertion itself.²³ Adjectives and verbs correspond to what Russell calls *concepts*, among which he distinguishes *predicates* from *relations*.²⁴ A predicate is a term that is indicated by an adjective; a relation is a term indicated by a verb.²⁵ Thus the proposition ⟨Plato admires Socrates⟩ is composed of three constituent terms, corresponding to the three words: ‘Plato’ and ‘Socrates’ indicate things, ‘admires’ indicates a concept, in particular a relation. The proposition ⟨Plato is human⟩ contains a thing (Plato) and two concepts: a predicate (*humanity*) and a relation (*being*).²⁶

Every term is a logical subject, and predicates and relations are no different.²⁷ Hence there are propositions whose subjects are predicates or relations (e.g. ⟨*humanity* is a predicate⟩ or ⟨*greater than* is a relation⟩). In the sentences expressing such propositions, we find phrases that are substantive in character but which have predicates or relations for their meaning. Russell describes such substantives as ‘derived from adjectives or verbs, as humanity from human or sequence from *follows*’ (*PoM*: §46, 42), but insists that he is ‘not speaking of an etymological derivation, but of a logical one’ (*PoM*: §46, 42).

²² Note that an assertion is not the same as a propositional function: propositional functions contain variables, assertions (in general) do not.

²³ They may be part of an assertion.

²⁴ Although not unrelated, Russell’s use of the word ‘concept’ should be sharply distinguished from Moore’s (also from Frege’s).

²⁵ I will only ever use ‘predicate’ to refer to non-linguistic entities, i.e. the correlates of adjectives or ‘predicate expressions’.

²⁶ These distinctions are discussed throughout *PoM*, ch. IV. Russell is aware that it may seem odd to treat the copula as a relation. He discusses it (without resolution) at *PoM*: §53, 49, pleading that: ‘it is so hard to know exactly what is meant by *relation* that the whole question is in danger of becoming purely verbal’.

²⁷ Similarly for propositions: when one proposition is the subject of another it occurs in unasserted form, as a propositional concept.

Concepts have the peculiar property that they may occur in propositions in more than one manner. Take the predicate *humanity*. It may occur in a proposition as part of the assertion, as in ⟨Socrates is human⟩; but may also occur as the subject of an assertion, as in ⟨humanity characterises Socrates⟩. Things, on the other hand, can only occur as the subject of an assertion. The distinction between things and concepts may therefore be illustrated as follows. The proposition ⟨Socrates is human⟩ can be analysed as subject and assertion in only one way²⁸: Socrates as subject, the rest of the proposition as assertion. The proposition ⟨*humanity* characterises Socrates⟩ on the other hand, can be analysed such that *humanity* is the subject, or that Socrates is the subject, or that both are. On this basis, Russell argues that the two propositions, though equivalent, are distinct:

In ⟨Socrates is human⟩, the notion expressed by *human* occurs in a different way from that in which it occurs when it is called *humanity*, the difference being that in the latter case, but not in the former, the proposition is *about* this notion. (*PoM*: §48, 45, angle-brackets added)²⁹

In the simple cases (i.e. those not involving denoting, which I discuss presently), a proposition is always about at least one of the terms that occurs in it. By distinguishing those constituent terms that a proposition is about from those that it is not about, Russell is able to characterise the distinction between the two modes of occurrence as follows. If a proposition *p* is about a given constituent term *t*, then *t* occurs in *p* as *subject*.³⁰

Russell has rather helped himself to the notion of *aboutness* here; and it would appear difficult to characterise aboutness without relying on the notion of *subject of a proposition*. But let us grant that the idea is sufficiently intuitive to be acceptable. Then it is the distinguishing feature of concepts, as opposed to things, that they may occur in a proposition not only as subject, but also in some other manner. Let us call that other manner of occurrence, occurrence *as concept*.

²⁸ This is the distinguishing feature of ‘subject-predicate’ propositions.

²⁹ Cf. *MTCA*: 53-4.

³⁰ Russell’s terminology is different. He would say that *t* ‘is a term of’ *p*, and that all of the other constituent terms occurring in *p* are not terms of *p*. But this is potentially misleading since those other terms are still terms, whether or not they are ‘terms of’ *p*.

The ultimate test of whether a term occurs in a proposition as subject or as concept concerns its substitutability conditions. We may substitute a term into a proposition to form a new proposition, but that term must have the same mode of occurrence as the term it replaces or the resultant complex will either fail to be propositional, or, if still propositional, will be differently structured. If a term may be substituted for *any term at all* (thing or concept), such that the resultant proposition has the same structure as the original, then that term occurs *as subject*. If, when substituting a term, the structure of the proposition is preserved only if the term is substituted for a concept, then that term occurs *as concept*. Extending our terminology, we may take the position in a proposition occupied by a term occurring as subject to be a ‘subject-position’. Similarly, the position in a proposition occupied by a term occurring as concept may be called a ‘concept-position’.

Let us take an example. In the proposition ⟨Socrates is human⟩, Socrates can be substituted for any term at all to yield a proposition of the same structure: substituting a thing, ⟨Plato is human⟩ is a true proposition; substituting a concept, ⟨insanity is human⟩ is a false proposition; both propositions have the same structure as the original. Socrates occurs in ⟨Socrates is human⟩, therefore, as subject (or in a subject-position). But compare the following substitutions for *human*. Substituting a concept we get, say, ⟨Socrates is ugly⟩, a proposition of the same structure. But substituting a thing yields, say, ⟨Socrates is Plato⟩, which, given the sense that *is* has here (i.e. predicative), is either not a proposition at all (since Plato cannot be asserted of anything), or, if we alter the sense of *is* (to the ‘is-of-identity’), is a proposition of a different structure (the original proposition was of subject-predicate form: the new one is relational). *Human* cannot therefore be substituted (while retaining the original propositional structure) for a thing, but only for another concept: that is to say, it occurs in ⟨Socrates is human⟩ as concept (or in a concept-position).³¹

³¹ A complicating factor should be noted here, though it does not materially affect any point I wish to make. In standard relational propositions, the relata occur as subject. In ⟨Plato admires Socrates⟩, for example, the two relata of *admires* occur as subject. This being so, if—as Russell sometimes seems to have done—one takes the ‘is’ of predication to be a relation, one may feel pressured to claim that *human* occurs as subject in ⟨Socrates is human⟩. Yet Russell would be adamant that *human* is not a logical subject of the proposition. Something has to give. Klement (2004a: 104-05) claims that Russell takes *is* to be a special kind of relation: one for which one relatum occurs as concept. He does not, however, provide any reference to support the interpretation. My own feeling is that Russell takes *human* to occur as concept and simply fudges the issue of whether *is* is a relation or not.

Consider now the proposition $\langle \textit{humanity characterises Socrates} \rangle$. *Humanity* and *human* are, Russell maintains, the exact same term. The differences between the $\langle \textit{humanity characterises Socrates} \rangle$ and $\langle \textit{Socrates is human} \rangle$ are a consequence of the fact that *human* (= *humanity*) occurs differently in each. Thus Russell describes predicates and relations as ‘capable of [a] curious twofold use’ (*PoM*: §48, 45). This ‘twofoldedness’ is demanded by fact that whatever *is* is a term. Predicates and relations must be able to occur as the logical subject of a proposition, on pain of contradiction.³²

2.4. The unity of the proposition again

Russell is, in this period, continually moved by the fear of a certain kind of contradiction—not the famous contradiction, the Russell paradox, but a different one. He describes it as ‘the contradiction always to be feared, where there is something that cannot be made a logical subject’ (*PoM*: §74, 76)³³. It threatens the claim, which for Russell is axiomatic, that *whatever is, is a term* (since every term is a logical subject of some proposition).³⁴

Frege’s problems with the concept *horse* (Frege 1892b) may be seen as an instance of this ‘contradiction always to be feared’. In Frege’s system, while one may say that ‘the city Berlin is a city’, and thereby express a Thought³⁵ about a certain city, one may not say that ‘the Fregean concept *horse* is a Fregean concept’ and thereby express a thought about a certain Fregean concept. By Frege’s lights, ‘the Fregean concept *horse*’ is a proper name; but proper names, on his view, only ever refer to objects, never to Fregean concepts. Frege puts this down to ‘an awkwardness of language’ (Frege 1892b: 185):

By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a [Fregean] concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me halfway – who does not begrudge a pinch of salt. (Frege 1892b: 192)

³² Similarly for propositions: in having both asserted and unasserted forms, propositions are ‘twofold’.

³³ Cf. *PoM*: §52, 48.

³⁴ Once the theory of denoting concepts is introduced, the mantra *whatever is, is a term* is seen to be false, as Russell introduces *objects*, strange combinations of terms, which are not themselves terms. This, he comments, leads to ‘grave logical problems’ (*PoM*: §58, 55n.).

³⁵ I use capitalisation to signal Frege’s technical notion of a Thought (*Gedanke*).

For Russell, on the other hand, this supposed awkwardness of language is a serious logical problem: ‘If there can be something which is not an object, then this fact cannot be stated without contradiction; for in the statement, the something in question becomes an object’ (Russell to Frege, 24 June 1902, in Frege 1980: 134). The implication is that there cannot be something which is not an object (in Russellian parlance, whatever is, is a term).

This particular Russell-Frege debate is of special interest here because of its relation to the question of propositional unity. For Frege there are objects and Fregean concepts, and these play fundamentally distinct roles (Fregean concepts being incomplete (*ungesättigt*) and essentially predicative). For Russell, a (Russellian) concept is a term with the potential for a ‘curious twofold use’: it may occur in a proposition as either subject or, predicatively, as concept. It is this ‘twofoldedness’ that Russell invokes in explanation of propositional unity. He calls our attention to the two modes of occurrence that a relation may have, calling this the distinction between a ‘relation in itself’ and a ‘relation actually relating’ (*PoM*: §54, 49) (a ‘relating-relation’). In every proposition there occurs a relation actually relating. When a relation occurs in this mode, it relates the other terms, uniting them as a proposition. If we imagine the mode of occurrence of that same relation changed, however, the proposition will be transformed from an asserted proposition to an unasserted proposition (a propositional concept). We can illustrate the transformation at the linguistic level by comparing ‘Plato admires Socrates’ with ‘Plato’s admiration of Socrates’: the former expresses a proposition, the latter a propositional concept. Thus Russell writes:

when [a relation] occurs as [concept], it actually relates, but when it occurs as [subject] it is the bare relation considered independently of the terms it relates. [. . .] Owing to the way in which the [relation] actually relates the terms of a proposition, every proposition has a unity which renders it distinct from the sum of its constituents. (*PoM*: §55, 52)

Superficially at least, Russell’s view is distinct from Frege’s.³⁶ It is distinct from Moore’s in that, although unity is explained by the presence of a relation in a proposition, the distinction between the possible modes of occurrence of that relation

³⁶ When fully examined, it is less clearly distinct from Frege’s than is immediately apparent. For a detailed discussion of the relations between the respective positions see Gaskin 2008 (especially §29).

allows for the distinction between a proposition and a mere aggregate of terms. A proposition contains a relation occurring as concept (a relating-relation), a mere aggregate of terms does not. Moreover, the relation of the relating-relation to the terms that it relates is explained in terms of the relating-relation's mode of occurrence; hence no further relation need be posited, and Bradley's Regress cannot get underway.³⁷

Russell's explanation of the problem of unity is only as good as his explanation of the connection between the relating-relation and the relation in itself. Generalising, what is required is an explanation of the relation between the two modes of occurrence that a concept may have in a proposition.³⁸ We saw above that Russell can only explain the distinction between occurrence as subject and occurrence as concept in terms of a proposition's being *about* some term, and suggested that the notion of aboutness cannot be explained without recourse to the notion of the subject of a proposition. While we may decide to grant Russell the intuitive notion of aboutness, this can only help him to explain the *difference in function* between a term occurring as subject and a term occurring as concept. But what is required at this point is an explanation of the *identity* of a particular term occurring as subject and that same term occurring as concept. Occurring as concept, a relation may have the property of uniting a collection of terms into a proposition: occurring as subject, that relation does *not* have the property of uniting the collection into a proposition. How, then, are the two identical, if the one has a property which the other lacks?

This problem may not be intractable³⁹, but Russell offers no explanation of it. The status of a term occurring as concept, and its connection with that same term occurring as subject, lies at the heart of Russell's metaphysics of propositions, but is shrouded in mystery. The relation of the two modes of occurrence will be of great importance in subsequent chapters.

3. The Theory of Denoting Concepts: Epistemological Considerations

Onto the framework surveyed above we must now superimpose the theory of denoting concepts. It is here that the issues in play in *OD* begin to come to the fore.

³⁷ The relating-relation stands in certain relations to each of the terms it relates; but these relations are external to the proposition in question. Cf. *PoM*: §55.

³⁸ A similar issue occurs in the Gray's *Elegy* Argument.

³⁹ Though I think it probably is.

Words have meaning, says Russell, in that they stand for, or indicate, something other than themselves. This kind of meaning—call it ‘linguistic meaning’—is in a sense *accidental*: take any word you like, and it is the case that that string of letters could have stood for something other than what it actually does. Unless we are essentialists about word-meaning (and Russell was not), the word ‘Socrates’ could have indicated something other than Socrates. Of course, once the word has been introduced into a linguistic community, it is plausible to suppose that it names Socrates if it names anything at all (if it named something else it would be a different word). But once established, it names Socrates independently of any facts about, or properties of, Socrates. Words, for Russell, have linguistic meaning because they are endowed with it by a linguistic community. In and of themselves, words are inert and meaningless.

However Russell acknowledges another kind of meaning, which we might call ‘logical meaning’. It is easiest to understand logical meaning through its opposition to linguistic meaning. If an entity *e* has linguistic meaning, then, though intrinsically meaningless, it has been endowed with a meaning by some agents. If *e* has *logical* meaning it is meaningful independently of anybody’s endowment of meaning upon it: that is to say, it is intrinsically meaningful. Since Russell takes logic to be concerned with propositions (rather than sentences), and since propositions do not contain words, Russell concludes that linguistic meaning is ‘irrelevant to logic’ (*PoM*: §51, 47). Where logic is concerned with meaning, it must be meaning in the other sense—the logical sense. The entities that have logical meaning are denoting concepts:

such concepts as *a man* have meaning in another sense [i.e. in the logical sense]: they are, so to speak, symbolic in their own logical nature, because they have the property which I call *denoting*. That is to say, when *a man* occurs in a proposition (*e.g.* (I met a man in the street)), the proposition is not about the concept *a man*, but about something quite different, some actual biped denoted by the concept. (*PoM*: §51, 47, angle-brackets added)⁴⁰

⁴⁰ In this quotation Russell misrepresents his own view. The proposition is not about some actual biped on his view, but about some strange combination of all men. See §4.2.1 below.

Denoting concepts thus mark a significant departure from the framework. A proposition may contain a term in subject-position without being *about* that term. Where this is so, that term is sure to be a denoting concept.

Why introduce a theory of denoting concepts at all? What does the theory do? There are two respects in which a theory of denoting is crucial, one logical, the other epistemological. On the logical side, a theory of denoting constitutes an account of generality: an account, that is, of the possibility of propositions (and so of thought) about some collection of entities specified by way of a common property, rather than enumeration. For Russell, working on the reduction of the truths of mathematics to truths of logic, a coherent theory of generality was indispensable: mathematics and logic abound with variables, and are therefore shot through with generality. On the epistemological side, one notes that our thoughts and judgements extend beyond just those entities with which we have direct acquaintance. The theory of denoting concepts offers an explanation of our ability to entertain the propositions involved in such judgements: the propositions in question might be *about* a certain entity (with which one lacks acquaintance) without *containing* that entity.

Commentators—notably Levine (1998) and Makin (2000)—often suggest that Russell introduced the theory of denoting concepts in response to epistemological concerns regarding infinite classes. In this section I outline these epistemological concerns and illustrate how the theory of denoting concepts solves them. In §4 I argue that this ‘epistemological view’ is mistaken, rests on an equivocation, and that the true explanation sheds far more light on the development of Russell’s philosophy.

3.1. Infinite classes and the problem of aboutness

Around 1900, Russell came to accept Georg Cantor’s theory of the infinite. Cantor’s theory predicts the existence of (an infinity of) infinite classes. According to a commonly espoused view, for Russell to accept that there could be infinite classes required a modification of the framework described in §2.

Russell describes terms as *whatever may be objects of thought* (PoM: §47, 43). It does not follow from this alone that every term is, in principle, a potential object of acquaintance. Nonetheless, this view is commonly attributed to Russell. Peter Hylton,

for instance, claims that acquaintance ‘carries with it no constraints’ and that ‘Russell [in *PoM*] seems to assume that every term (in his sense) is a possible object of acquaintance’ (Hylton 1990: 245). Interestingly, no textual support is provided for the claim. The closest we come to support is a claim Hylton makes—regarding not Russell, but G. E. Moore—that, on Moore’s view, the world is ‘transparent to the intellect’ and ‘made up of the objects of thought’ (Hylton 1990: 137) (bearing in mind, of course, that the objects of thought exist independently of being thought about).

The view that whatever is, is a possible object of acquaintance is one which, I would suggest, does not sit happily with the kind of realism that Russell and Moore defended during the period in question in this thesis. Terms must certainly not be *defined* as possible objects of acquaintance: to define them by way of possible cognitive relations to minds would be to downplay their radical independence from human thought. Indeed, from a certain perspective, to attribute this view to Russell would be to cast him as a kind of anti-realist. Levine, for instance, points out the difference between the view which Hylton attributes to Russell in the previous paragraph, and the brand of realism endorsed by Thomas Nagel (1986). On this conception, what there is may be *so* radically independent of our conceptual capacities as to be, in principle, impossible for us to conceive of or to have acquaintance with. According to Levine, the early Russell (i.e. the Russell committed merely to the framework surveyed in §2 above) is not, by Nagel’s lights, a realist (Levine 1998: 438). Levine’s view is that while the framework surveyed above, sans the theory of denoting concepts, is anti-realist in the sense currently at issue, the theory of denoting concepts ‘results from a move to the sort of realism which enables us to countenance entities with which we cannot be acquainted’ (Levine 1998: 439).

Levine attributes to Russell the following view, which (we shall assume for now that) he held before—but not after—adopting the theory of denoting concepts:

- (L1) We can be acquainted with any entity; there is no entity with which we cannot, in principle, be acquainted. (Levine 1998: 418)⁴¹

⁴¹ This is Levine’s principle ‘R₃’

But having accepted Cantor's theory of the infinite, Russell now accepts that there are entities with which, in principle, we could not be acquainted: infinite classes. Being mere mortals, limited and finite of mind, we cannot have acquaintance with an infinite collection such as the class of real numbers. Acquaintance with every element of an infinite class is, in principle, impossible: *a fortiori* acquaintance with the infinite class is impossible too.

For the purposes of mathematics, and so of logicism, one must have some way of dealing with infinite collections. One might, for instance, wish to say of an infinite class that it is larger than some other class, that it is countable, or whatever. Suppose we wish to consider a proposition about some infinite class. From the framework as described thus far, a proposition is about whatever terms occur in subject-position therein. But given (L1), no proposition that the likes of us could grasp could contain an infinite class in subject-position. Thus the problem arises: *how it is possible that there may be propositions which we entertain, but which are about an entity (or entities) with which we cannot be acquainted?* I shall refer to this problem as 'the problem of aboutness'. If Levine's view is that the theory of denoting concepts is introduced in the course of rejecting (L1), his view is that the theory is introduced in the course of solving the problem of aboutness.

The problem of aboutness extends beyond just those entities with which one cannot, in principle, have acquaintance, to those with which one cannot, *practically speaking*, have acquaintance, and to those with which one might well (practically speaking) have been acquainted, but with which one, *as it has turned out*, does not have acquaintance. Consider the propositions expressed by (2) and (3).

- (2) All men are mortal
- (3) Each dog in that room is asleep.

Acquaintance with all men and each dog in that room are not required in order to entertain the propositions expressed by these sentences. The question then, is what marks the case of infinite classes out as distinct from cases such as these. Presumably the difference is that in the case of infinite classes, acquaintance is *in principle impossible*, while in the others it is merely *contingently absent*. But this difference does

not, at least concerning the issues at hand, amount to very much. The philosophical underpinnings of Russell's logicism must explain not only how it is possible to think about infinite classes, but also the possibility of thought about any other collection of terms (including very large or epistemologically remote classes). One might suggest, alternatively, that the fact that acquaintance with an infinite class is *in principle* impossible, indicates that the problem is not epistemological, but metaphysical. According to this suggestion, the problem stems from the fact that there cannot *be* a proposition whose subject is an infinite class. But if that were the case, then Russell would hold that there cannot be infinite classes at all, since whatever is, is the subject of a proposition.⁴² (Notice that although the problem could not then arise for infinite classes, with respect to very large or epistemologically remote classes it would remain). The problem at hand must, therefore, be considered as epistemological, in which case it does not only concern infinite classes.

3.2. A solution to the problem of aboutness

Suppose I took a walk this morning and bumped into Jones on the street. I might report this encounter by saying:

(4) I met a man.

Suppose you are not acquainted with Jones and, in any case, do not know which particular man I met this morning. Upon hearing my utterance of (4), you surely entertain some proposition. But which? It cannot be a proposition containing Jones, for you are unable to entertain any such proposition (not being acquainted with him). The phrase 'a man', as it occurs in (4), does not then indicate Jones. Yet it seems obvious that it does not indicate any other *thing* either (for there is no thing other than Jones that I met), and so, by a process of elimination, one might infer that it must indicate a concept.⁴³

What mode of occurrence does this concept have in the proposition expressed by (4), i.e. $\langle 4 \rangle$? If it occurred as subject, then $\langle 4 \rangle$ would be about the concept *a man* (in the

⁴² I explore a related suggestion in §4 below.

⁴³ I take it that this inference does not commit one to the claim that one met a concept, but only that what occurs in the proposition is a concept.

same way that $\langle \textit{happiness is good} \rangle$ is about the concept *happiness*). But as Russell points out, this cannot be the case here:

If I say “I met a man,” the proposition is not about *a man*: this is a concept which does not walk the streets, but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank-account or a public-house and a drunken wife. (*PoM*: §56, 53)⁴⁴

But on the other hand, we may also rule out the possibility that the concept occurs as concept. If it did, $\langle 4 \rangle$ would not be about *a man*—neither the concept, nor any man—all. For a proposition is only about those terms that occur in it as subject, not those occurring as concept.

The solution is that the concept *a man* occurs in $\langle 4 \rangle$ as subject, but that, because *a man* is a denoting concept, the proposition is not about *a man*, but about something else: ‘such concepts as *a man* [. . .] are, so to speak, symbolic in their own logical nature, because they have the property which I call *denoting*’ (*PoM*: §51, 47). *A man* is therefore a denoting concept—it has meaning in the logical sense, which is what is meant by describing it as ‘symbolic in [its] own logical nature’. Russell tells us that a concept *c* is a *denoting concept* if, when ‘it occurs in a proposition, the proposition is not *about* [*c*], but about a term connected in a certain peculiar way with [*c*]’ (*PoM*: §56, 53). This ‘certain peculiar connection’ is the logical relation of *denoting*.⁴⁵ Consider, for example, sentence (5):

(5) The inventor of the clarinet was German.

If the unmodified view were still in place (i.e. if *the inventor of the clarinet* were considered a concept *simpliciter* rather than a *denoting* concept) we should say that $\langle 5 \rangle$ was about the concept *the inventor of the clarinet*. But clearly it is not about the concept, but about whoever invented the clarinet—namely, Johann Christoph Denner.

⁴⁴ See n. 40 above. In such passages as this Russell unintentionally implies that a correct analysis will yield the result that the proposition is about the very man I met. Ultimately his analysis yields the result that although I met a particular man, the proposition is not *about* him, but about a strange combination of all men. These ‘combinations’ are discussed briefly in §4.2.1.

⁴⁵ For reasons that will only become clear in §4, it is not obviously and un-controversially the case that a proposition containing a denoting concept is about a *term* connected with it in a peculiar way.

Crucially, one may entertain $\langle 5 \rangle$ even if one has never heard of, and is unacquainted with, Denner. Denoting concepts, then, are concepts indicated by phrases of the form ‘all F ’, ‘every F ’, ‘any F ’, ‘an F ’, ‘some F ’, and ‘the F ’, which share the common characteristic that, when they occur in a proposition, the proposition is not about them themselves, but about whatever it is that they denote. The denoting phrase ‘The inventor of the clarinet’ indicates the denoting concept *the inventor of the clarinet*, which denotes Denner. For clarity, henceforth I shall distinguish denoting concepts by enclosing them in angle brackets (‘ \langle ’ and ‘ \rangle ’). Thus, for example, I shall say that the sentence ‘the teacher of Plato is wise’ expresses the proposition $\langle \text{the teacher of Plato} \rangle$ is wise), which contains the denoting concept $\langle \text{the teacher of Plato} \rangle$, which denotes Socrates.

An interesting question—entirely neglected in the literature—is why Russell should think of denoting concepts as *concepts* at all. I have suggested the reason above: the denoting phrase in (4) does not seem to indicate a *thing*: it doesn’t stand for Jones, and if I met any *thing* at all, it was certainly Jones. But denoting concepts have one of the hallmarks of things, namely they cannot occur in a proposition as concept. One can no more predicate $\langle \text{a man} \rangle$ of Socrates than one can predicate Plato of him. In the proposition $\langle \text{Socrates is } \langle \text{a man} \rangle \rangle$, the ‘is’ is the ‘is-of-identity’ on pain of nonsensicality. Denoting concepts are, I think one ought to conclude, not concepts at all, but things—albeit things of a peculiar stripe. Denoting concepts are certainly closely related to concepts *simpliciter*. For one thing, they are obtained from them (as described in §4). For another, just as a proposition containing a concept is not usually about that concept, so a proposition containing a denoting concept is not usually about that denoting concept. (Though it *may* be. If this were impossible, denoting concepts would not be possible logical subjects, and so not terms at all: denoting concepts, like concepts *simpliciter* are assumed to be capable of a curious ‘twofold’ use. This twofold use is not identical to the twofold use of concepts *simpliciter*, however. It will be the subject of investigation in the Gray’s *Elegy* Argument.) Nevertheless, despite these similarities, the key fact remains: denoting concepts cannot occur as concept. This being so, all bets are off: denoting concepts are not concepts.

The most obvious distinction between denoting concepts and all other terms is that they *denote*. For this reason they are aptly described as ‘aboutness-shifters’ (Makin 2000: 17-8): when a denoting concept occurs in a proposition, it ‘shifts’ aboutness from

itself, onto something else. As a crude analogy, compare the use of mirror to look at an object around a corner. One looks at the mirror, but observes the object. The mirror, one might say, shifts aboutness from itself onto the object in a manner roughly analogous to that in which a denoting concept shifts aboutness from itself onto its denotation. In this way, a proposition may be about a term which does not occur in it. For instance (5) is about Denner, without his occurring therein. In the case of a sentence such as (6) exactly the same explanation occurs.

(6) Every number has a successor.

⟨6⟩ contains a denoting concept, <every number> (indicated in (6) by ‘every number’), which denotes the infinite class of numbers; it does not contain every single number, though the denoted class does. So in general we have a solution to the problem of aboutness. The ability to apprehend a proposition about an entity (or entities) with which one is unacquainted is to be explained by the fact that it is not always the case that what a proposition is about is to be found among its constituents. Whenever one entertains a proposition about an entity with which one is unacquainted, it may be inferred that the proposition includes among its constituents a denoting concept—with which one *is* acquainted—that denotes the entity that the proposition is about. Thus Russell writes:

With regard to infinite classes, say the class of numbers, it is to be observed that the [denoting] concept *all numbers*, though not itself infinitely complex, yet denotes an infinitely complex object. This is the inmost secret of our power to deal with infinity. An infinitely complex concept, though there may be such, can certainly not be manipulated by the human intelligence; but infinite collections, owing to the notion of denoting, can be manipulated without introducing any concepts of infinite complexity. (*PoM*: §72, 73)

The introduction of the theory of denoting concepts means that Russell now distinguishes two ways in which a proposition may be about an entity *e*.

(Ab1) A proposition may be about an entity *e* in virtue of containing *e* (in subject-position).

(Ab2) A proposition may be about an entity *e* in virtue of containing a denoting concept *d* (in subject-position), such that *d* denotes *e*.

Now the theory of denoting concepts is in a certain respect similar to Frege's theory of *Sinn*.⁴⁶ But it should be noted that, since Frege applies his distinction between *Sinn* and *Bedeutung* across the board, not only to denoting phrases, he has no use for (Ab1).⁴⁷ The most fundamental difference between the Russellian and Fregean positions is, however, that, for Russell, the theory of denoting concepts is intended as an account of generality.⁴⁸ The theory is a logical theory which offers an account of the logico-metaphysical fact that some propositions are about entities that do not occur in them. The Fregean theory of *Sinn* is largely independent of the account of generality. For Frege, *Sinn* is primarily a cognitive notion; whereas for Russell, the theory of denoting concepts is primarily an account of generality.

4. The Theory of Denoting Concepts: The Logical Genesis View

It is undeniable that Russell takes the theory of denoting concepts as a response to the epistemological worries raised by the problem of aboutness. But that does not in itself provide a justification of the view, notably endorsed by Levine and Makin, that Russell introduced the theory in response to that particular problem. Makin rightly emphasises the connection between the theory of denoting concepts and the mathematical context in which the theory is presented, but he wrongly supposes that 'the chief consideration for recognizing denoting as a logical constant is its role in relation to infinite classes' (Makin 2000: 14). The truth of the matter is, I think, that Russell introduces the theory as a general consequence of his views on predicates and on classes.

The epistemological view does find textual support. Levine points us in the direction of the following passage from *PoM*:

⁴⁶ Frege (1892a). Since they are familiar, I leave the terms '*Sinn*' and '*Bedeutung*' un-translated.

⁴⁷ Obviously (Ab1) and (Ab2) are not given in Fregean terms, but the sense in which they might be applied to Frege's position is clear enough.

⁴⁸ I am careful to use the phrase 'account of...' rather than 'explanation of' generality. As I discuss at the end of the chapter (§4.2.2), Russell did not think that the theory of denoting concepts *explained* generality. My view is thus different from that of Hylton (2005d: 202-03; 2005c: 165).

Indeed it may be said that the logical purpose which is served by the theory of denoting is, to enable propositions of finite complexity to deal with infinite classes of terms [...]. (*PoM*: §141, 145)

This passage is not from the chapter of *PoM* in which Russell introduces denoting. So if we take it at face-value, it seems strange that Russell should make so little mention of infinite classes in the chapter on denoting. The only mention in that chapter comes in §60 where Russell says that the possibility of denoting a class is ‘highly important, since it enables us to deal with infinite collections’ (*PoM*, §60, 58).⁴⁹ One sentence out of a thirteen page chapter is not a lot. Moreover, the epistemological view offers little account of the fact that the theory of denoting constitutes Russell’s treatment of generality in *PoM*. So I suggest that it should come as no surprise if a better explanation of the origins of the theory of denoting concepts can be found.

I will call the alternative view that I endorse the ‘logical genesis view’, since it takes very seriously Russell’s statement that: ‘The notion of denoting may be obtained by a kind of logical genesis from subject-predicate propositions’ (*PoM*: §56, 54). I will begin by setting out the logical genesis view. I then argue that the epistemological view, as expounded by Levine, rests on an equivocation which, once resolved, leaves the epistemological view poorly motivated.

4.1. *The logical genesis view*

Russell takes it that the simplest propositions are of subject-predicate form (*PoM*: §57, 54)—the following, for example:

- ⟨7⟩ ⟨A is⟩
- ⟨8⟩ ⟨A is one⟩
- ⟨9⟩ ⟨A is human⟩

⟨7⟩-⟨9⟩ (according to *PoM*: §57, 54) are equivalent, respectively, to:

- ⟨10⟩ ⟨A is <an entity>⟩
- ⟨11⟩ ⟨A is <a unit>⟩

⁴⁹ *Infinity* (not infinite classes) is also mentioned at *PoM*: §59, 58.

⟨12⟩ ⟨*A* is <a man>⟩

However ⟨10⟩-⟨12⟩ ‘are not identical with the previous ones since they have an entirely different form’ (*PoM*: §57, 54). For one thing, Russell says, in the latter group but not the former, *is* is the only term which does not occur as subject.⁵⁰

That is, in (say) ⟨9⟩, *A* occurs as subject, and *is* (a relation) and *humanity* (a concept) occur as concept⁵¹; while in ⟨12⟩, *A* and <a man> occur as subject, while only *is* occurs as concept.

Russell then writes:

A man, we shall find, is neither a concept nor a term, but a certain kind of combination of certain terms, namely of those which are human. And the relation of Socrates to *a man* is quite different from his relation to humanity.
(*PoM*: §57, 54)

A correct understanding of this passage is important. When Russell says that *a man* is neither a concept nor a term, he means that *what is denoted by* the denoting concept <a man> is neither a concept nor a term (he has misapplied his convention of using italics to distinguish denoting concepts). Thus Russell’s position in the above quotation is that the relation of Socrates to *humanity* is very different to his relation to the ‘combination’ of all humans. His relation to *humanity* is, let us suppose, that of *falling under (the predicate)*: his relation to the combination of all humans is something which is, if not identical, then analogous to *class-membership*.

Russell takes it that the relation of mutual implication between ⟨9⟩ and ⟨12⟩ is, in some sense, rooted in the fact that the predicate in ⟨9⟩, *humanity*, is intimately linked to a class-concept, *man*, which ‘give[s] rise’ to the class of men (*PoM*: §57, 54). That is, ⟨9⟩ implies ⟨12⟩ because the predicate in ⟨9⟩ gives rise to the class of men (in virtue of the relation between the predicate *human* and the class-concept *man*), and the denoting concept in ⟨12⟩, i.e. <a man>, denotes (as we shall see below) a kind of combination of

⁵⁰ Notice the slip here. In ⟨7⟩ *is* is the only term which does not occur as subject. I owe this observation to Murali Ramachandran.

⁵¹ Cf. n. 31 above.

the members of that class. The notion of ‘giving rise’ is unclear, as is Russell’s distinction between a predicate and a class-concept. The two issues are closely linked.

As to the latter, Russell claims (*PoM*: §57, 54) that it will be convenient to distinguish a class-concept from a predicate (speaking of non-linguistic entities). Thus *human* is a predicate while *man* is a related class-concept. In general, a class-concept is associated with a certain predicate. Both ‘give rise’ to the same class, the members of which all ‘have’ (i.e. satisfy) the predicate. Nonetheless, Russell acknowledges his uncertainty as to the distinction between predicates and class-concepts, admitting that it is ‘perhaps only verbal’ (*PoM*: §58, 56).

Russell’s thought is that for any member x of a given class of which the class-concept is u , there is a proposition of the form $\langle x \text{ is } \langle a \ u \rangle \rangle$. So to take his own example, the predicate *human* gives rise to a class, of which the class-concept is *man*; and for any member x of that class, we may truly assert that $\langle x \text{ is } \langle a \ \text{man} \rangle \rangle$. The example is unfortunate, since $\langle x \text{ is } \langle a \ \text{human} \rangle \rangle$ (using the predicate rather than the class-concept) would appear to do just as well. But perhaps we can see the ‘verbal’ distinction more clearly in the following example. Take the proposition, $\langle \text{Descartes doubts} \rangle$. *Doubts*—the predicate—gives rise to a class (*doubters*) of which the class-concept is *doubter*. Hence there is a value of x for which $\langle x \text{ is } \langle a \ \text{doubter} \rangle \rangle$ is true. So in general we may distinguish a predicate from a class-concept, and both from what Russell calls the ‘concept of a class’ (*PoM*: §67, 67). In his example, *human* is the predicate, *man* the class-concept, and *men* the concept of the class; in our example, *doubts* is the predicate, *doubter* the class-concept, and *doubters* the concept of the class. (I think we must agree with Russell’s suggestion that the distinction, such as it is, is merely verbal.)

What is it for a predicate⁵² to give rise to a class? In one respect, it is terms (in general)—rather than predicates (in particular)—that give rise to classes. From the extensional standpoint:

it is not predicates and denoting that are relevant [to the genesis of classes], but terms connected by the word *and*, in the sense in which this word stands for a

⁵² What is said here about predicates applies also to class-concepts.

numerical conjunction. Thus Brown and Jones are a class, and Brown singly is a class. This is the extensional genesis of classes. (*PoM*: §68, 67)

From the *intensional* standpoint, however, it is predicates that give rise to classes:

We may, then, imagine a kind of genesis of classes, through the successive stages indicated by the typical propositions “Socrates is human,” “Socrates has humanity,” “Socrates is a man,” “Socrates is one among men.” [. . .] [E]very subject-predicate proposition gives rise to the other three equivalent propositions, and thus every predicate (provided it can be sometimes truly predicated) gives rise to a class. This is the genesis of classes from the intensional standpoint. (*PoM*: §68, 67)⁵³

We may now begin to plot the course of the logical genesis view. Every predicate gives rise to a class. This is effectively to say that, for every predicate and its associated class-concept u , ‘ x is a u ’ is a propositional function.⁵⁴ That is, for every predicate p , there exists a proposition asserting that something is a member of the class to which p gives rise (though there need not be a *true* proposition of this form⁵⁵).

It will be characteristic of class-concepts—as opposed to terms in general—that when substituted for u in a proposition of the form $\langle A \text{ is a } u \rangle$, the result will always be a proposition (*PoM*: §58, 56). But what mode of occurrence does the class-concept u have in the proposition $\langle A \text{ is a } u \rangle$? When a term occurs as subject, it may be substituted for any other term such that the structure of the proposition remains unchanged. But in our case, ‘ u has a restricted variability if the formula is to remain a proposition’ (*PoM*: §58, 56). Thus u does not occur as subject. This shows that ‘a u ’⁵⁶ is not analysable into ‘a’ and ‘ u ’ (‘a u ’—or ‘some u ’, ‘any u ’, etc.—is not a function of u , for, for that to be the

⁵³ The parenthetical caveat is important. In its absence, it would follow that, for every predicate, there is a term to which the predicate may be truly ascribed. This would lead to a larger ontology than is necessary. It would also follow that there is no null-class. In itself, this is unproblematic (for it also follows from the extensional genesis of classes that there is no null-class). But what is problematic is that it would entail that there are no null class-concepts and no null concepts of a class—and these are necessary for symbolic logic (*PoM*: §73).

⁵⁴ To this point I have said nothing about propositional functions. I shall say more in Chapter Two. For now, a propositional function may be thought of as a function with n variables which, when substituted for terms, yield a proposition. Russell: ‘We may explain (but not define) [the notion of a propositional function] as follows: ϕx is a propositional function if, for every value of x , ϕx is a proposition, determinate when x is given’ (*PoM*: §22, 19).

⁵⁵ Cf. n. 53 above.

⁵⁶ I use inverted commas here to mention an entity (not a phrase). I would like to use my angle bracket notation, for ‘a u ’ is a denoting concept (i.e. $\langle a u \rangle$), but to do so would beg a question.

case, *u* would have to occur as subject in the functional complex; but, as proved by its restricted variability, *u* does not occur as subject). This leads Russell to say that a denoting phrase ‘consists always of a class-concept preceded by one of the above six words [i.e. ‘all’, ‘every’, ‘any’, ‘a’, ‘some’, ‘the’] or some synonym of one of them’ (*PoM*: §58, 56). Given his account of linguistic meaning, a denoting phrase is meaningful in virtue of standing for a term: the denoting phrase ‘a *u*’ stands for the denoting concept $\langle a\ u \rangle$. Thus we arrive, having started with a predicate, at a denoting concept. Notice that while not once has it been necessary to mention infinite classes, the notions of *class* (in general) and *denoting concept* are intimately linked. Let us now turn back to the epistemological view, and try to locate where it goes wrong.

4.2. Against the epistemological view

The epistemological view—at least as it is expounded by Levine (and Levine is the only commentator to offer the issue serious attention)—rests on an equivocation. Levine argues that Russell introduces the theory of denoting concepts for the purposes of rejecting a principle he had previously held, namely:

- (L1) We can be acquainted with any entity; there is no entity with which we cannot, in principle, be acquainted. (Levine 1998: 418)

The claim is that once Russell accepted Cantor’s theory of infinity, he held that we cannot, in principle, be acquainted with infinite classes, and thus that (L1) is false. However, the word ‘entity’, as it occurs in (L1), requires disambiguation. My claim is that, in *PoM*, Russell would only have rejected (L1) if ‘entity’ is read as ‘object’. If it is read as ‘term’, on the other hand, I hold that Russell would not necessarily have rejected the claim.⁵⁷ This does not, as it may appear to, commit me to the problematic claim that Russell thought we *could* be acquainted with infinite classes; this is because of an ambiguity in the notion of *class*. But before moving on to classes, we begin with the distinction between terms and objects.

⁵⁷ I have no general objection to the use of ‘entity’ in discussions of Russell. Levine uses it consistently (see his 1998, 2001, 2004 and 2005), and I shall too, in later chapters. The problem with its employment in (L1) is local, so to speak.

4.2.1. Terms and objects

‘Entity’ is not a technical expression in *PoM*, but Russell uses ‘term’ as synonymous with it. A term, as we have seen, is ‘whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*’ (*PoM*: §47, 43). It is, Russell says, ‘the widest word in the philosophical vocabulary’ (*PoM*: §47, 43) since ‘anything [...] that can be mentioned, is sure to be a term’ (*PoM*: §47, 43). However, later in the next chapter Russell writes in a footnote that he will ‘use the word *object* in a wider sense than *term*’ (*PoM*: §58, 55n.), and comments that ‘The fact that a word can be framed with a wider meaning than *term* raises grave logical problems’ (*PoM*: §58, 55n.).

‘Objects’, for Russell, are the denotations of certain denoting concepts, namely all denoting concepts except those expressed by definite descriptions. Taking a finite class composed of $a_1, a_2, a_3 \dots a_n$, Russell offers the following explanations (see *PoM*: §§59-61; note that parenthetical remarks are not quoted, but are intended to be explanatory; all else *is* quoted):

- (1) *All a*’s denotes a_1 and a_2 and $\dots a_n$.
(Any such object Russell calls a ‘numerical conjunction’.)
- (2) *Every a* denotes a_1 and denotes a_2 and \dots and denotes a_n .
(Any such object he calls a ‘propositional conjunction’.)
- (3) *Any a* denotes a_1 or a_2 or \dots or a_n , where *or* has the meaning that it is irrelevant which we take.
(Any such object he calls a ‘variable conjunction’.)
- (4) *An a* denotes a_1 or a_2 or \dots or a_n , where *or* has the meaning that no one in particular must be taken, just as in *all a*’s we must not take one in particular.
(Any such object he calls a ‘variable disjunction’.)
- (5) *Some a* denotes a_1 or denotes a_2 or \dots or denotes a_n , where it is not irrelevant which is taken, but on the contrary some one particular *a* must be taken.
(Any such object he calls a ‘constant disjunction’.)

The details of the various modes of combination of terms to form objects are complicated, but we need not investigate them further for our purposes. The important point is that these five kinds of object are not themselves terms, but are *combinations* of terms:

There is, then, a definite something, different in each of the five cases, which must, in some sense, be an object, but is characterized as a set of terms combined in a certain way, which something is denoted by *all men*, *every man*, *any man*, *a man* or *some man*; and it is with this very paradoxical object that propositions are concerned in which the corresponding concept is used as denoting. (*PoM*: §63, 62)

Interpreting (L1) as a claim about objects, it is certainly the case that Russell does not hold it in *PoM*. For instance, consider the denoting concept <all numbers>. This denotes a combination (a numerical conjunction) of 1, 2, 3, . . . [*ad infinitum*]. Such a combination is not a possible object of acquaintance (at least for finitely-minded beings).

If we consider (L1) to be a claim about terms, Russell's view is less clear. We can however go some way towards recovering his view from the text. He says that 'a class [. . .] or anything else that can be mentioned, is sure to be a term' (*PoM*: §47, 43): but what about *infinite* classes? If a finite class may be a term, then there is no reason to suppose that an infinite class cannot also (classes should be not classified—at least not at the logical or metaphysical level—by reference to our capacities to manipulate them). So if a finite class with n members may be a term, then so may one with n^n members, and so may one with $(n^n)^n$ members, (and so on indefinitely). Why then, may not an indefinitely large class, be a term? The fact that we could not name or think directly about such a class is simply irrelevant at the logical or metaphysical level. To make a distinction based on such considerations would seem to be, firstly, poorly motivated, and, secondly, distinctly anti-realist (why should a realist suppose that what there is depends on what we may have acquaintance with?). So the distinction between finite and infinite classes does not seem to be especially relevant here. The really relevant distinction is between the notions of *class-as-one* and *class-as-many*.

4.2.2. *Classes-as-one and classes-as-many*

Classes, as we have seen, may be approached *extensionally* or *intensionally*. To define a class extensionally, one enumerates its members. Infinite classes cannot—by humans at least—be defined extensionally, and so, from a practical point of view, must be defined intensionally. To define a class intensionally, one specifies the concept of the class. In

these ways we may think of a class either as a collection of terms, or as a unified whole. The former is the class-as-many: the latter the class-as-one.

A class also, in one sense at least, is distinct from the whole composed of its terms, for the latter is only and essentially one, while the former, where it has many terms, is [. . .] the very kind of object of which *many* is asserted. The distinction of a class as many from a class as a whole [i.e. *as one*] is often made by language: space and points, time and instants, the army and the soldiers, the navy and the sailors, the Cabinet and the Cabinet Ministers, all illustrate the distinction. (*PoM*: §70, 68)

In a class-as-many, the member terms have some kind of unity, but ‘just so much unity as is required to make them many, and not enough to prevent them from remaining many⁵⁸’ (*PoM*: §70, 69). The unity of the class-as-many is clearly different to (and weaker than) the unity of the proposition. Given the intractable difficulties Russell faces in regard to propositional unity one could be forgiven for doubting the possibility of an adequate explanation of the very strange and weak unity of the class-as-many. But let us leave that worry to one side.

Might it be the case that the class-as-one and the class-as-many are one and the same, perhaps the same collection under different modes of presentation? Russell argues not, as follows. If a concept *c* denotes denotation *d*, then, if *d* is identical to *d'*, *c* denotes *d'*. The concept *classes of all rational animals* denotes, Russell says, the human race *as-one*, and is different from the concept *men*, which denotes the human race *as-many* (*PoM*: §74, 76). If the human race *as-one* were identical to the human race *as-many*, *classes of all rational animals* would denote the human race *as-many*, and *men* would denote the human race *as-one*. Since, on Russell’s view, this is not the case (*PoM*: §74, 76), the class-as-one and the class-as-many are distinct:

it is [. . .] correct, I think, to infer an ultimate distinction between a class as many and a class as one, to hold that the many are only many, and are not also one. (*PoM*: §74, 76)

⁵⁸ That is, not enough to make them *one*.

So a class-as-one is distinct from its associated class-as-many.⁵⁹ And classes-as-many are not terms, but objects:

Thus *man* is the class-concept, *men* (the concept) is the concept of the class, and men (the **object** denoted by the concept *men*) are the class. (*PoM*: §67, 67, bold emphasis added)

And it is because they have that strange kind of unity—enough to make them *many* (rather than simply *disparate*), but not enough to make them *one*—that objects are described as ‘very paradoxical’ (*PoM*: §62, 62).⁶⁰ Compare propositions: a proposition is a collection of terms, but is itself also a term. This is because a proposition has a special kind of unity, generated by a concept occurring as concept. A proposition may be a ‘logical subject’. For instance, one may entertain the proposition ⟨⟨Socrates is mortal⟩ is true⟩, a proposition directly about a subordinate proposition. But compare a similar judgement concerning a class-as-many. Russell writes:

In such a proposition as “*A* and *B* are two,” there is no logical subject: the assertion is not about *A*, nor about *B*, nor about the whole composed of both, but strictly and only about *A* and *B*. (*PoM*: §74, 76-7)

A very strange claim, but Russell’s point is this. The proposition in question is true, yet the assertion ‘are/is two’ is not true of *A* (for *A* is one), nor of *B* (for *B* is one too), nor yet of the whole composed of *A* and *B* (for this whole—*qua* whole—is also one⁶¹). The assertion is true of *A* and *B* considered *together but severally* (paradoxical as this sounds). Russell says, ‘Thus it would seem that assertions are not necessarily *about* single subjects, but may be about **many** subjects’ (*PoM*: §74, 77, bold emphasis added).

This consideration is used by Russell in disarming a familiar contradiction: ‘can we now avoid the contradiction always to be feared, where there is something that cannot be made a logical subject?’ (*PoM*: §74, 76). The worry is that the class-as-many might not be a possible logical subject of a proposition; for when it is made the logical subject

⁵⁹ I investigate the relation of class-as-one and its associated class-as-many in Chapter Two.

⁶⁰ At least plural combinations are paradoxical in this way (and they may be paradoxical in other ways too). Russell sometimes claims that a denoting concept such as <a man> denotes an ambiguous man. Is an ambiguous man many or one? I don’t know. If not, then—though obviously problematic—he is not paradoxical in the above sense.

⁶¹ Cf. ‘every whole is one’ (*CP3*: 35).

of a proposition, that proposition appears to have a single subject, yet the class-as-many is, by definition, not single, but many. Russell resolves the threatened contradiction by claiming that a class-as-many *can* occur in subject-position in a proposition, but that in such cases the proposition will be about the members of that class considered *together but severally*⁶². This means that the proposition $\langle A \text{ and } B \text{ are two} \rangle$ is not exactly about each member of the class (indicated by the phrase ‘ $A \text{ and } B$ ’) *individually*, but not exactly about the two considered *together* either. (Compare the previous indented quotation above.)

Transposing from a finite class to an infinite one, the exact same considerations apply. Suppose we have an infinite class composed of terms $t_1, t_2, t_3 \dots$ [*ad infinitum*]. We will denote the (transfinite) cardinal number of the class by the symbol ‘ \aleph ’. Suppose then that there is a true proposition such as $\langle t_1 \text{ and } t_2 \text{ and } t_3 \text{ and } \dots$ [*ad infinitum*] are \aleph ’. This proposition is not about any term individually (for it is not true of t_1 (say) that it is \aleph); but nor is it about the whole composed of t_1, t_2, t_3, \dots [*ad infinitum*] either (for it is not true of this whole—*qua* whole—that it is \aleph either). In fact, the proposition is about t_1 and t_2 and t_3 and \dots [*ad infinitum*] considered together but severally. So Russell manages to avoid his feared contradiction in cases involving both finite and infinite classes-as-many.

Now this purported resolution is not, perhaps, very compelling. The objects upon which it rests (i.e. numerical conjunctions, classes-as-many) are highly suspect. But two points are clear. The first is that the contradiction which agitates Russell is only threatened if the class-as-many is a term (for the contradiction expressly concerns terms *qua* logical subjects). The second is that Russell’s ‘resolution’ is in fact a denial that the contradiction arises here. For the class-as-many is not, like a genuine term, a *single* logical subject, but irreducibly plural. The class-as-many is an *object* not a term. Russell’s point, in effect, is that while the notion of a term which cannot be a logical subject of a proposition is contradictory, this restriction does not apply to objects. So when Russell said that ‘a class [. . .] is sure to be a term’ (*PoM*: §47, 43), he had in mind classes-as-one, not classes-as-many.

⁶² This turn of phrase is not Russell’s but mine. I fail to see what else he could possibly mean by saying that $\langle A \text{ and } B \text{ are two} \rangle$ is not about A , nor B , nor the whole composed of A and B , but ‘strictly and only about A and B ’ (*PoM*: §74, 77).

A class-as-one is a term, and presumably may, relatively un-problematically, be the logical subject of a proposition. Consider, for instance:

⟨13⟩ ⟨*Men* is a class⟩

Moreover, Russell says, speaking of proposition ⟨14⟩, that it ‘explicitly contains the class as a constituent’ (*PoM*: §68, 67).

⟨14⟩ ⟨Socrates is one among men⟩

Here, the very fact that one may entertain ⟨14⟩ indicates that ‘class’ should be read as ‘class-as-one’ in the quoted claim; for although I believe that Socrates is one among men, I am certainly not acquainted with every single human (all the more so since Russell takes the human race to include all past, present, and future humans (*PoM*: §62, 62)). If the class-as-many occurred in ⟨14⟩, it would be, practically-speaking, impossible for mere mortals like us to apprehend that proposition.⁶³ But since Russell would not have accepted that we cannot entertain ⟨14⟩, one should infer that he held that the class-as-one occurs therein.

But now consider the proposition

⟨15⟩ ⟨343 is one among numbers⟩

By parity of form, this proposition also *explicitly* contains the class as a constituent. But in this case, the class is an infinite class, *numbers*. Given that one may, in spite of one’s finitude, entertain the proposition in question, and given the preceding discussion, does that not entail that one may be acquainted with an infinite class-as-one? If so, then reading (L1) as about *terms*, there is no need to suppose that Russell rejected it in *PoM*.

⁶³ There would also be a problem with null class-concepts. If ⟨Socrates is one among men⟩ was equivalent to, say, ⟨Socrates is one among: Socrates, Plato, Pele, Hilary Clinton, Henry VIII, Lulu . . . [and so on, listing every human that ever was, is, or will be]⟩, then what proposition would ⟨Socrates is one among unicorns⟩ be equivalent to?

A null-class, a unit-class, a very large class, and an infinite class, may all—taken as-one—be objects of acquaintance.⁶⁴

At this point one might mount an objection, claiming that ⟨15⟩ does not contain the infinite class as a constituent at all (either as-one or as-many), but, rather, contains some concept that denotes it. Indeed Russell does say that, for example, the concept *men* denotes (the object) men (*PoM*: §67, 67). If this objection is correct, then Russell’s claim that ⟨14⟩ ‘explicitly contains the class as a constituent’ (*PoM*: §68, 67) was erroneous. But moreover, if this is the case, Russell was mistaken in claiming that a class may be a term *at all*, considered either as-many or as-one. For if ⟨14⟩ does not contain the class-as-one, then it is difficult to see how ⟨13⟩ could. But if ⟨13⟩ does not contain a class-as-one, it is difficult to see how *any* proposition could contain a class-as-one. And if no proposition could contain a class-as-one, then no class-as-one is a possible logical subject or a possible object of thought—in short classes-as-one are not terms; and since classes-as-many are not either, Russell was wrong to say that ‘a class [. . .] is sure to be a term’ (*PoM*: §47, 43).

There is something to be said for this objection to my interpretation, in as much as it highlights the inconsistencies of Russell’s presentation (and, in all likelihood, his view).⁶⁵ Moreover, as shall emerge in the next chapter, I fully accept that the status of the class-as-one is highly problematic for Russell. Nevertheless, I do not find the objection compelling since when Russell says ‘a class [. . .] is sure to be a term’ (*PoM*: §47, 43) I take him at his word.

Interestingly, even if the objection were sound, this could only provide further evidence in favour of the logical genesis view over the epistemological view. What is at issue here is the connection between classes and concepts, the objection being that I have not accurately represented Russell’s view of that connection. Whether the interpretation I have defended above or the view touted in the objection is correct, Russell’s presentation (if not his view) is inconsistent in *some* respect: classes-as-one are either terms (as I propose) or they are not (as in the objection), yet both sides of the

⁶⁴ Obviously using ‘objects’ in a non-technical sense here.

⁶⁵ We should certainly not be frightened to attribute inconsistencies to Russell in *PoM*, for they are not uncommon. To cite one example, in Chapter VI of Part I, we are told that there is no null-class; by Chapter X, however, there is.

debate have presented some evidence in favour of their interpretation. But wherever the inconsistency is ultimately located, an important point emerges. If it is true that classes are not terms, then this applies to all classes, infinite or not. And if, on the other hand, classes-as-one *are* terms, then there is as much reason to suppose that one may be acquainted with infinite ones as with finite ones—for an infinite class-*as-one* is simply *one*, not (infinitely) many.

Returning to Levine's principle (L1), in the light of the preceding discussion I see no reason to suppose Russell committed to the claim that there are any *terms* with which we cannot, in principle, be acquainted.⁶⁶ Some *objects*—some classes-as-many—are, in principle, beyond possible acquaintance, but that is another matter. The theory of denoting concepts is not, I submit, introduced to explain how we may form propositions about infinite classes, but as a consequence derived, by a kind of logical genesis, from subject-predicate propositions. The theory of denoting concepts certainly shows its worth in relation to infinite classes-as-many, for, in principle, *we* could not form propositions about them in any other way. But—practically speaking—we could not form propositions about, say, the class-as-many of *men* in any other way either, for that class just has too many members. So I conclude that the theory of denoting concepts was introduced in the context of a general account of the basic notions of logic—*predicate* and *class* in particular—and was not specifically addressed to the epistemological worry raised by the problem of aboutness, contra the epistemological view.

This is not to say that Russell's acceptance of Cantor's theory was not of great importance in sparking his renewed investigation into the nature of classes.⁶⁷ Worries about the use of the notion of infinity in the logicist project clearly exercised Russell; and he described the problems of infinity and of infinite number as 'soluble by a correct theory of *any*' (*PoM*: §179, 188). But this is merely the identification of an application for the theory of generality.⁶⁸ It is perfectly compatible with the logical genesis view,

⁶⁶ A possible exception to this would be an infinitely complex concept: but Russell is agnostic as to the existence of such terms (*PoM*: §73, 73).

⁶⁷ He said, for instance: 'I was led to [the paradoxes] by considering Cantor's proof that there is no greatest cardinal number' (*MPD*: 75).

⁶⁸ In a similar vein, Hylton (1996: 39) describes Russell as *invoking* the theory of denoting concepts in response to the problems of infinity. If he *invokes* it, then it is already extant independently of the problem of aboutness.

and is not what the epistemological view attests. The general thrust (though not the detail) of the view that I have defended is advocated by other commentators⁶⁹, though they do not provide any support for it. I have sought to justify the view since, in this thesis, it will be important to recognise that *the theory of denoting is, for Russell, primarily an account of the possibility of propositions of generality*.⁷⁰

This is *not* to say that the theory of denoting concepts is intended as an *explanation* of generality. Such a view is advocated by Hylton (2005d: 201-03; 2005c: 165). On his view, whereas in *OD* Russell *assumes* generality (by taking the variable as ‘fundamental’ (*OD*: 42)), in *PoM* Russell attempts to *explain* it in terms of denoting concepts. The theory of descriptions is able to replace the theory of denoting concepts without loss of explanatory value in this respect because, Hylton claims, the theory of denoting concepts *fails* as an explanation of generality. So Russell loses nothing by giving it up. But in fact, generality is assumed in *PoM*. Although the variable is not taken as fundamental (but treated as a denoting concept) *generality* is still assumed in virtue of the relation of *denoting*—the relation between a denoting concept and its denotation—being taken as a primitive logical relation. It is the relation that is key, and it is presupposed in both the *OD* and *PoM* accounts of the variable. As Hylton himself acknowledges (e.g. Hylton 1990: 256), the logical relation of denoting remains in *OD*, in the form of the relation between the variable and its values.

Recognising the role of the theory of denoting concepts as an account of the possibility of propositions of generality is important in a study of this kind, for it fits in far more happily than the epistemological view with the general tenor of Russell’s work in the period. Alasdair Urquhart, in his introduction to *CP4*, contends that:

Most of the very voluminous secondary literature on Russell’s Theory of Descriptions discusses it in isolation from its setting in the enterprise of the logical derivation of mathematics; the resulting separation of the logical and the mathematical aspects of denoting is foreign to Russell’s own approach. (Urquhart, in *CP4*: xxxii)

⁶⁹ Hylton 2005d: 195-96; Wahl 2007: 12; Landini 2009: 285.

⁷⁰ Note that this distinguishes the theory of denoting from the theory of *Sinn*.

It is an advantage of the position I am defending—the logical genesis view—that it ties in extremely closely with Russell’s own account of his work after the completion of *PoM*. He wrote, for example:

When *The Principles of Mathematics* was finished, I settled down to a resolute attempt to find a solution of the paradoxes. [. . .] Throughout 1903 and 1904, my work was almost wholly devoted to this matter, but without any vestige of success. My first success was the theory of descriptions, in the spring of 1905. (*MPD*: 79)

The logical genesis view enables us to begin to set the theory of descriptions—which is rooted in the theory of denoting concepts—on firmly logico-mathematical ground, as Urquhart suggests one ought to.

By focussing discussion on Cantor’s theory and on infinite classes, the epistemological view does place Russell’s discussion of denoting in a mathematical context. But, given that *PoM* is a work in the philosophy of mathematics, it would be nothing short of a miracle if any account of the theory of denoting concepts didn’t manage to set it in *some* kind of mathematical context. However, at root, on the epistemological view, the theory of denoting concepts centres on the problem of aboutness; that problem is primarily epistemological, and is not, in and of itself, inherently mathematical. On the logical genesis view, however, denoting is an issue for Russell precisely *because of* its connection with (what he took to be) a central notion of mathematics, namely the notion of *class*.

The connections between the theory of denoting concepts, the account of generality, and the theory of classes having been established, I shall turn in the next chapter to the question of why Russell, in the period 1903-05, looked to the theory of denoting in his attempt to solve the paradoxes that bear his name.

2. Denoting and the Paradoxes

1. Introduction

To one acquainted only with Russell's later reflections upon the production of *Principia Mathematica* (*PM*), it might seem that the discovery of the theory of definite descriptions and the search for a solution to the paradoxes at the heart of the logicist project were wholly unconnected. Russell writes, for example:

Throughout 1903 and 1904, I pursued will-o'-the-wisps and made no progress. At last, in the spring of 1905, a different problem, which proved soluble, gave the first glimmer of hope. The problem was that of descriptions, and its solution suggested a new technique. (*MMD*: 13)

However, passages such as this misrepresent the relation between the paradoxes and the question of descriptions and denoting. The idea that, in a serendipitous turn of fate, Russell happened upon 'the first step towards overcoming the difficulties which had baffled me for so long' (*Auto*: 155) may have a certain charm, but is nonetheless demonstrably false. The theory of descriptions and the paradoxes are, in Russell's philosophy at least, bound together. Indeed evidence from Russell's own pen shows that he took the question of denoting to be relevant to the paradoxes:

in April 1904 I began working at the Contradiction again, and continued at it, with few intermissions, till January 1905. I was throughout much occupied by the question of Denoting, which I thought was probably relevant, as it proved to be. (Grattan-Guinness 1977: 79)

Of course thinking the question of denoting to be 'probably relevant' is not exactly a ringing endorsement of its close connection to the paradoxes. But a stronger connection can be established.

The roots of the theory of descriptions may be traced, via the Gray's *Elegy* Argument (GEA), back to the theory of denoting concepts from *PoM*. In Chapter One I suggested that the theory of denoting concepts is an account of generality, and best understood as arising by a kind of 'logical genesis' from the nature of predicates. In arguing for the interpretation we began to see the very close connection between

denoting concepts and classes, and glimpsed the connection of the ‘logical genesis’ of denoting concepts and the ‘intensional genesis’ of classes.

In this Chapter I develop the connection between the paradoxes and the theory of denoting. Russell’s investigations into the theory of denoting in the period 1903-05 (i.e. between the publications of *PoM* and *OD*) were a central part of his attempt to resolve the paradoxes. Generalising, we find in the literature two common claims: that the roots of the theory of descriptions lie in the GEA; and that Russell’s investigations in the period 1903-05 centred on the search for a solution of the paradoxes. Both of these claims are true, but what is wanting is an explanation of the connection between them. In the absence of such an explanation we can merely gesture towards an understanding of Russell’s work in this period. My aim in this chapter is to begin to bring that connection out into the light. By the end of Chapter Five the connections will, I hope, be clear.

In §2 I look at Russell’s formulations of the paradoxes. I demonstrate—by appealing to a confusion in his presentation—that Russell was attempting to steer a course between a purely extensional and purely intensional view of classes. The confusion stems from his distinction between ‘classes-as-many’ and ‘classes-as-one’. In Chapter One I suggested that the status of classes-as-one is unclear. Here I shall develop this thought, arguing that the class-as-one occupies a kind of limbo between its associated class-as-many and the propositional function or predicate that gives rise to that class-as-many. Though I do not claim that Russell was explicitly aware of the issue in the terms in which I present it, I argue that the relation between a class-as-one and its associated class-as-many involves a blurring of the lines between extension and intension. I take this complex interplay of extension and intension to be a constant theme in Russell’s work on the paradoxes and denoting in the period 1903-05.¹

In §3 I look at Russell’s strategies for tackling the paradoxes. I am not so concerned to set out the precise details of the various theories he attempted to formulate, but rather consider the standards that Russell demanded of a satisfactory solution. In illustrating

¹ The connections cannot be brought out with precision as Russell never formulated the relevant concepts or positions with precision. For further evidence of the extension/intension interplay, in both Russell and Frege, see Gaskin (2008: 165ff).

Russell's concerns I argue against a widespread understanding of his commitment to the doctrine of the unrestricted variable.

There are, I shall suggest, two main points of connection linking the paradoxes and the theory of denoting: the question of 'empty' denoting phrases, and the question of the relation between *that which denotes* (i.e. a denoting concept or, as in Russell's post-*PoM* works, a denoting complex or meaning) and *that which is denoted*. In §3.3 I address the former, the question of empty denoting phrases. Almost inevitably now, 'empty denoting phrases' leads us to think of 'the present king of France', 'the golden mountain', 'the round-square' and so on. These immediately raise questions in ontology or the philosophy of language. Such questions certainly interested Russell, but I seek to emphasise his more strictly logical concerns in this area, and in particular the connection to the paradoxes. The connection is not merely such that one can construct it after the fact, but was rather a live question for Russell at the time.

§4 turns to the relation between 'meaning' and 'denotation'. I set out Russell's 'theory of meaning and denotation' and examine its relation to the theory of denoting concepts. Russell toyed, in his unpublished manuscripts from 1903-05, with some significant developments which, had they held, would have rendered the theory of meaning and denotation unarguably distinct from the theory of denoting concepts. I show that the theory of meaning and denotation upon which Russell eventually settles differs from the theory denoting concepts only with respect to very minor modifications and changes of terminology, leaving the basic structure intact. Unlike most other commentators, I attempt to demonstrate both how the theory of meaning and denotation is relevant to Russell's investigations into the paradoxes *and* how his investigations led to the GEA.² To this end I set out Russell's distinction—key to the GEA—between 'entity-occurrence' and 'meaning-occurrence' (§4.2).

The investigation into the paradoxes is, I claim, the glue that holds together the theory of denoting concepts, the theory of meaning and denotation, the GEA, and the theory of descriptions. The theory of descriptions and the paradoxes are inextricably bound. The aim of this chapter is to establish the thematic ties.

² Notable exceptions include: Rodriguez-Consuegra (1989; 1992); Wahl (1993); Landini (1998a; 1998b); and Levine (2005).

2. The Paradoxes: Intension and Extension

Russell reports that the ‘logical honeymoon’ he had enjoyed since his encounter with the work of Peano at the International Congress of Philosophy in 1900 came to an abrupt end in the spring of 1901 with the discovery of the Contradiction.³

In *PoM* the paradox is stated in a variety of forms. For a statement of the basic problem one can do worse than turn to Russell’s statement of it from 1908:

Let w be the class of all those classes which are not members of themselves. Then, whatever class x may be, ‘ x is a w ’ is equivalent to ‘ x is not an x ’. Hence, giving to x the value w , ‘ w is a w ’ is equivalent to ‘ w is not a w ’. (*ML*: 59)

Unpacking this somewhat, the point is as follows. Some classes are members of themselves and some are not. The class b of badgers, for example, is a class, not a badger; as such, if we enumerate the members of b , we will list badgers, but not b itself: b is not a member of itself. The class c of classes, on the other hand, *is* a member of itself: it contains all and only those things that have the property of *being a class*, and c certainly has that property, so c is included among its own members. Taking w to be the class of all those classes which, like b , are not members of themselves, we note that:

- If w is a member of itself, then—as such—it is not a member of w (for w is composed of all and only those classes which are not members of themselves).
- If w is *not* a member of itself, then—in virtue of that very fact—it *is* a member of the class of classes which are not members of themselves, i.e. of w .

In that case, w is a member of itself just in case w is not a member of itself: $w \in w \leftrightarrow w \notin w$.

The first statement of the paradox in *PoM* is given in terms of predicates (*PoM*: §78). Some predicates are, and some are not, truly predicable of themselves (ϕ is truly predicable of itself if ‘ ϕ is ϕ ’ expresses a true proposition). *Being a badger*, for

³ In fact it seems that Russell did not realise the full impact of the paradoxes until he communicated them to Frege in 1902. On this see Kilmister 1984: 89 and G. H. Moore 1995: 234–35. I have not included an account of the origins of the paradox. On this see Coffa 1979, Garciadiego 1992, and G. H. Moore 1995.

example, is not a badger, so it is not truly predicable of itself. *Being a predicate*, on the other hand, in virtue of being a predicate, *is* truly predicable of itself. Consider, then, the following predicate: *not being truly predicable of oneself* (call it ' ϕ '). Is ϕ truly predicable of itself? Since ϕ is truly predicated of some term x just in case x is not truly predicable of itself, i.e.

$$(1a) \quad \phi(x) \leftrightarrow \neg x(x)$$

ϕ will be truly predicable of itself just in case it is not truly predicable of itself. To illustrate, substituting ϕ for x in (1a) yields a contradiction:

$$(1b) \quad \phi(\phi) \leftrightarrow \neg \phi(\phi)$$

The first paradox, the paradox of classes, is sometimes thought of as the *extensional counterpart* of the second, the paradox of predicates (which is thought of as the *intensional counterpart* of the former).⁴

This may, in general, be a useful distinction to draw, but it is potentially misleading to the student of *PoM*. When, in *PoM*, the paradox of classes is given, it is given not as above simply in terms of w , but in terms of Russell's distinction between the class-as-one and the class-as-many:

A class as one may be a term [i.e. member] of itself as many. Thus the class of all classes is a class; the class of all the terms that are not men is not a man, and so on. Do all the classes that have this property form a class? If so, is it as one a member of itself as many or not? If it is, then it is one of the classes which, as ones, are not members of themselves as many, and *vice versa*. (*PoM*: §101, 102)

(Russell is guilty of a gross slip here. The question is not, as he writes, 'Do all the classes that have [the property of being, as ones, members of themselves as many] form a class?', but rather, as he surely intends, 'Do all the classes that have the property of

⁴ E.g. Stevens 2005: ch. 2.

not being, as ones, members of themselves as many form a class?') I call this version of the paradox the 'mixed' paradox.⁵

Though nobody seems to remark upon it⁶, it should be noted that the statement of the paradox given above is inconsistent with other of Russell's claims regarding the nature of classes. Russell feels the pull of two claims: that a class is a kind of plural object, and that a class should be the kind of entity which can be counted or of which an assertion could be made:

Is a class which has many terms to be regarded as itself one or many? Taking the class as equivalent simply to the numerical conjunction "A and B and C and etc.," it seems plain that it is many; yet it is quite necessary that we should be able to count classes as one each, and we do habitually speak of *a* class. (*PoM*: §74, 76)

It is, after all, axiomatic for Russell that whatever is (has being), can be the logical subject of a proposition (*PoM*: §47).

Thus we must distinguish two senses in which the notion of *class* is to be employed: on the one hand a class is a term, a logical subject, the kind of thing of which an assertion may be made. On the other hand, classes must be considered as 'numerical conjunctions', for it is only in this sense that *number* can be predicated of them:

All men, for example, denotes men conjoined in a certain way; and it is **as thus denoted** that they have a number. Similarly *all numbers* or *all points* denotes [*sic*] numbers or points conjoined in a certain way, and **as thus conjoined** numbers or points have a number. Numbers, then, are to be regarded as properties of classes. (*PoM*: §109, 113, bold emphasis added)

When Russell says that numbers are 'to be regarded as properties of classes' he clearly means 'of classes-as-many'. Or again:

A class also, in one sense at least, is distinct from the whole composed of its terms, for the latter is only and essentially one, while the former, where it has

⁵ 'Mixed' since it is, as I will show presently, intermediate between the extensional paradox of classes and the intensional paradox of predicates.

⁶ As far as I am aware.

many terms, is [. . .] the very kind of **object** of which *many* is to be asserted. (*PoM*: §69, 68, bold emphasis added)

Russell's use of 'object' here is deliberate, indicating that the class is to be taken as a numerical conjunction.⁷

But the difficulty is that the class-as-many and the class-as-one are *distinct*: they are not the same thing, not even the same thing under different modes of presentation or different modes of occurrence. Russell writes,

it is [. . .] correct, I think, to infer an ultimate distinction between a class as many and a class as one, to hold that the many are only many, and are not also one. (*PoM*: §74, 76)⁸

This complicating factor—that the class-as-many and class-as-one are distinct—is rarely discussed in the literature.⁹ This is a surprising omission since, as discussed below, it serves to illustrate the interplay of the notions of extension and intension in regard to Russell's position on classes and the paradoxes. That the issue is rarely discussed may be due to a misunderstanding regarding the motivation for positing the distinction between the two forms of class. Cocchiarella (1987: 22), for instance, suggests that Russell's coming to distinguish the two forms of class was a consequence of his coming to deny, in the wake of the paradoxes, that for every class-as-many there is an associated class-as-one. But as I have suggested, that Russell distinguishes the class-as-many and class-as-one is simply due to their having different properties—*number* is truly predicable only of classes-as-many, and is, as such, quite independent of the paradoxes.¹⁰

Holding to this line, Russell's statement of the mixed paradox will not do: a class α is not, as one, a member of *itself* as many—if there is an 'ultimate distinction' between a class-as-many and a class-as-one, then 'itself' is quite out of place. Contrary to Russell's statement of the mixed paradox, α -as-one *may not* be a member of itself as-

⁷ Cf. *PoM*: §58, 55n.

⁸ And this view is confirmed later on: the class-as-one is 'in any case not identical with the class as many' (*PoM*: §127, 132).

⁹ It is mentioned by Gaskin (2008: 168-69).

¹⁰ Dau (1986: 140), L. Linsky (1998: 634-35), and Oliver & Smiley (2005: 1041) all get this right, it seems to me.

many. In order to restate the mixed paradox in these terms, Russell must provide an account of the relation between the class-as-one and the class-as-many. Such an account is by no means impossible; but it is surprisingly difficult to give an account that does not blur the lines between extension and intension, as we shall see.

2.1. *Classes-as-many and as-one: the extensional and intensional standpoints*

PoM does not always present Russell's finalised position. For example, the views contained in the appendices are sometimes in tension with the line taken in the main body of the work.¹¹ Russell confesses to feeling 'little confidence in my present opinions', describing his conclusions as 'essentially hypotheses' (*PoM*: xxi). As regards classes, he can only comment that 'the contradiction [. . .] proves that something is amiss, but what this is I have hitherto failed to discover' (*PoM*: xxi). Neat characterisations of the paradoxes as, in this form extensional, in that form intensional, are therefore liable to pass over subtleties relevant to the development of Russell's thought.

In what follows we will consider only classes with more than one member. Null- and unit-classes are of tremendous importance to Russell's project, but they introduce complications which, for present purposes, may be put to one side. As a final preliminary we must introduce some notation. Lower case roman letters (a, b) will stand for terms. To speak of a class without explicit commitment to its being considered as-one or as-many I use early (in the alphabet) lower case Greek letters (α, β, γ): to specify whether the class is taken as-one or as-many I use superscripted '1' or 'M' (thus α^1 is distinguished from α^M). I use ϕ and ψ as predicate expressions; these in conjunction with x, y, z , and the 'hat', form expressions for propositional functions: thus $\phi\hat{x}$ is a propositional function yielding a proposition when a term is substituted for \hat{x} .

In Chapter One we spoke of classes as derived from predicates and class-concepts. (We may take predicates and class-concepts to be the same thing: Russell acknowledges that 'the distinction is perhaps only verbal' (*PoM*: §58, 56), and in what follows I shall use the two interchangeably.) To speak in this way is not *wrong*, but it simplifies

¹¹ The main body itself is not always consistent. Compare the treatments of the null-class at §73 and §106.

somewhat. Russell's official view is that classes are derived from propositional functions¹²; and in light of the paradoxes he is uncertain as to the relation between propositional functions and class-concepts. It might seem obvious that any propositional function $\phi\hat{x}$ must be associated with some predicate ϕ , and I think Russell felt the force of this article of common sense; but in the face of the paradoxes he felt moved to agnosticism as to the ontological status of such would-be class-concepts as *not being a member of oneself*, *not being a member of one's own extension*, *not being truly predicable of oneself* and so on. Though Russell deems the existence of such class-concepts questionable, he does not doubt the existence of the associated propositional functions (though the ontological status of propositional functions is uncertain¹³.) What positive claims Russell has to make of the relation between class-concepts and classes may be safely extended to the relation between propositional functions and classes—though not vice versa. That is, while Russell accepts that for any predicate ϕ there is an associated propositional function $\phi\hat{x}$, he does not accept that for any propositional function $\phi\hat{x}$ there is an associated predicate ϕ .

From the extensional standpoint, a class may be 'defined by the enumeration of its terms' (*PoM*: §68, 68) using the concept of 'and'¹⁴. Thus a class α may be defined as:

$$(\text{Df}_{\text{ext}}: \alpha) \quad a\text{-and-}b\text{-and-}c$$

(supposing that α has just three members, a , b , and c). On the other hand, from the intensional standpoint, a class is defined in terms of the propositional function which gives rise to it. Thus α might be defined as:

$$(\text{Df}_{\text{int}}: \alpha) \quad \{x: \phi\hat{x}\}$$

where ' $\{x: \phi\hat{x}\}$ ' is read 'the class of x such that x is/has ϕ '.

¹² Though he does not put matters in these terms in the sections of *PoM* discussed in Chapter One (which is why I followed his terminology there).

¹³ Cf. *PoM*: §85.

¹⁴ Russell attributes to *and* the ability to combine terms in such a way that they 'have [. . .] just so much unity as is required to make them many, and not enough to prevent them from remaining many' (*PoM*: §70, 69; cf. *PoM*: §§71, 130; see also Chapter One, §4).

It may be tempting to align the class-as-many with the class defined in terms of extension and the class-as-one with the class defined in terms of intension. This temptation should be resisted. Classes-as-one may, just as much as classes-as-many, be defined extensionally; and classes-as-many may, just as much as classes-as-one, be defined intensionally. Importantly, classes, both as-one and as-many, have extensional identity conditions. That is to say, they are ‘definite when their terms [i.e. members] are given’ (*PoM*: §66, 66). On the other hand, predicates, class-concepts, and propositional functions, though they may have an extension, are themselves intensional entities.

The class-as-one must also be distinguished, says Russell (*PoM*: §69), from its associated class-concept or propositional function, and for good reason: different class-concepts (or propositional functions) can define the same class (this serves to further emphasise that the identity conditions of classes are extensional).

When we come to examine the mixed paradox, the lines between the extensional and intensional standpoints begin to blur. We may construct a wholly extensional paradox of classes-as-one. Here the ‘punch-line’ will be:

$$(2) \quad \alpha^1 \in \alpha^1 \leftrightarrow \alpha^1 \notin \alpha^1$$

This is simply the paradox of classes as given above, quoted from *ML*. Now consider the mixed paradox. Since Russell has not noticed that his claim that α^1 may be a member of *itself*-as-many cannot be quite right, he presumably takes it that (3a) captures the paradox as he intends it.

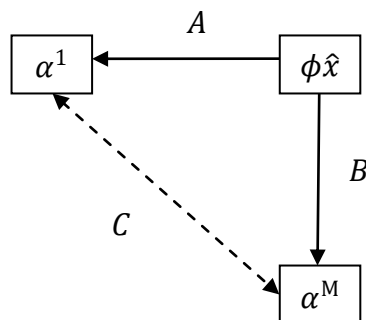
$$(3a) \quad \alpha^1 \in \alpha^M \leftrightarrow \alpha^1 \notin \alpha^M$$

If α^1 and α^M were identical (so that Russell’s ‘itself’ claim held true) then (3a) would be purely extensional. But since this is not so we must probe the relationship between α^1 and α^M .

Although they are non-identical, α^1 and α^M obviously do bear some rather intimate relationship. At heart, we might think, that relationship stems from their both being

derivable from the same propositional function. Suppose that that propositional function is $\phi\hat{x}$. Then $\phi\hat{x}$ ‘gives rise’ to both α^1 and α^M (this ‘giving rise’ relation cannot be the same in both cases). We might diagram the relations as follows:

*Fig. 1*¹⁵



The ‘giving rise’ relations A and B are perhaps not perfectly well understood, but that need not concern us unduly. The relation with which we are primarily concerned is C . We could characterise C in terms of A and B ; but it seems natural to suppose that the relation between α^1 and α^M is not, ultimately, mediated via A and B and $\phi\hat{x}$, but is independent of them. That is to say, α^1 and α^M are the very classes that they are by virtue of their containing as members just those terms that they do: their identity conditions are extensional and so no intensional aspect need come into their relation, one might suppose.

Not wishing to characterise the relationship between α^1 and α^M in such a way as to introduce intensional notions, what kind of purely extensional account of the relation could we provide?

To begin with, notice that given Russell’s general strategy for tackling the paradoxes, we cannot simply take the relation to be fundamental, a basic logical fact. Russell’s proposal for dealing with the paradoxes is to deny that all propositional functions determine a class-as-one (though all determine a class-as-many). This is

¹⁵ This form of diagram was first suggested to me by a similar one—though not one involving classes—in Demopoulos 1999.

equivalent to denying that for every class-as-many there is an associated class-as-one.¹⁶ The idea is to delineate and then explain which classes-as-many have an associated class-as-one. Now if the relation C is taken as a fundamental logical relation, then how could any class-as-many ever fail to bear it to some class-as-one? And of course it would not help to propose that relation C has a *sense*, i.e. a *direction*, namely *from* a class-as-one *to* a class-as-many. This, it might be thought, restricts the obtaining of C to those cases where both forms of class subsist. But this is clearly no great advance. Let C' be the converse of C , and now ask how any class-as-many could ever fail to bear C' to some class-as-one? Given that C is a fundamental relation, it is hard to deny that C' is too (insofar as C' is definable, i.e. in terms of C , C is equally definable in terms of C').

A more fruitful line is to suggest that C is to be characterised in terms of a class-as-one and a class-as-many *having the same members*. One might posit something like the following condition:

$$(C_i) \quad C(\alpha^1, \alpha^M) \leftrightarrow (x)(x \in \alpha^1 \leftrightarrow x \in \alpha^M)$$

That is, α^1 and α^M stand in relation C just in case any member of α^1 is a member of α^M and any member of α^M is a member of α^1 . But here we must be careful not to confound two senses of '∈'. The relation of a term to a class-as-one is not identical to the relation of a term to a class-as-many (*PoM*: §76). Thus we must say rather,

$$(C_{ii}) \quad C(\alpha^1, \alpha^M) \leftrightarrow (x)(x \in_i \alpha^1 \leftrightarrow x \in_{ii} \alpha^M)$$

using subscripts to distinguish the two *membership* relations. The mixed paradox then concerns the class-as-one of those classes-as-one which are not members of the class-as-many to which they stand in the C relation. Is this class-as-one a member of the class-as-many to which it stands in the C relation? We find that it is just in case it is not. That is:

$$(3b) \quad \beta^1 \in_{ii} \beta^M \leftrightarrow \beta^1 \notin_{ii} \beta^M$$

¹⁶ Cf. *PoM*: §101.

But there remain two complications in this analysis that re-introduce intensional elements into the supposedly extensional paradox. Firstly, there is no general characterisation of the relation \in_{ii} that does not necessitate appeal to intensional entities. Secondly, the class-as-one occupies a curious intermediate position, some way between the wholly extensional class-as-many and the wholly intensional class-concept (predicate). I deal with these in turn.

The class-as-many is irreducibly plural: it is not a single term. This, it will be recalled from Chapter One, required Russell to introduce *objects* in addition to terms: ‘The fact that a word can be framed with a wider meaning than *term* raises grave logical problems’, says Russell (*PoM*: §58, 55n.). We are never explicitly told what these ‘grave logical problems’ are, though they all seem to concern the so-called ‘contradiction always to be feared, where there is something that cannot be made a logical subject’ (*PoM*: §74, 76). Presumably the following is among their number.

The relation of class-membership is two-place, so that $z \in \gamma$ asserts a certain relation between z and γ . The relation \in_{ii} , we stipulated, was to hold between a term and a class-as-many; this was required in order for us to characterise the relation C holding between the class-as-one and class-as-many. But matters are not so simple, as Russell notes:

It is plain that, since a class, except when it has one term, is essentially many, it cannot be *as such* represented by a single letter: hence in any possible Symbolic Logic the letters which do duty for classes cannot represent the classes *as many*, but must represent either class-concepts, or the whole composed of classes, or some other allied single entities. And thus \in cannot represent the relation of a term to its class as many; for this would be a relation of one term to many terms, not a two-term relation such as we want. (*PoM*: §76, 78)

This is, in effect, to call into question our notation ‘ α^M ’. No such simple expression could do justice to the irreducible plurality of the class-as-many. In the passage Russell offers three alternatives. The expression ‘ α^M ’ might represent a class-concept; it might represent the class-as-one, i.e. α^1 ; or it might represent ‘some other allied single entity’. We may rule out the third as unclear and *ad hoc*. The second is plausible in some cases,

but can be of no use to us in our attempt to characterise C . If ' α^M ' represents α^1 , then C becomes:

$$(C_{iii}) \quad C(\alpha^1, \alpha^1) \leftrightarrow (x)(x \in_i \alpha^1 \leftrightarrow x \in_i \alpha^1)$$

which is plainly not what we want. That leaves us only the first option, namely that ' α^M ' represents a class-concept. A class-concept, however, is an intensional entity. It follows then, that there is no characterisation of the relation C that does not involve some kind of appeal to, or reliance upon, an intensional notion.

The second complication necessitating the re-introduction of intensional notions concerns the status of the class-as-one. It is clear that, for Russell, classes are, in the primary and most important sense, to be considered as-many.¹⁷ He says, for instance, that 'without a single *object* to represent an extension, Mathematics crumbles' (*PoM*: §489, 515, emphasis added). We should not be misled by the word 'single' here. The claim is that mathematics cannot get on without *objects*, and Russell inserts a footnote immediately after the word 'object', directing the reader to the footnote from §58 in which the word 'object' is introduced as having a wider sense than 'term'. At least part of the reason why mathematics cannot get on without classes-as-many is that the notion of *number* is applicable only where we have classes-as-many.¹⁸ This is plain in the case of classes having more than one member.¹⁹ But even as regards unit-classes, the same point holds. Following Frege in the *Grundlagen* (Frege 2007), Russell suggests that the sense in which any *term* (thus not *class-as-many*) may have a number (i.e. *one*) predicated of it is 'a very shadowy sense, since it is applicable to everything alike' (*PoM*: §128, 132). On the contrary, he claims, the sense in which a class-as-many is a *unit-class-as-many* is precise: 'A class u has one member when u is not null, and " x and y are u 's" implies " x is identical with y "' (*PoM*: §128, 132).²⁰ This is extendable in the familiar way for cardinalities greater than one. Since we must be able to say of a class that it is a member of another class, and since a class is, in its primary sense, a class-as-

¹⁷ In the literature this does not always seem to be recognised. Largely, I suspect, this is because the class-as-one, inasmuch as it is a *single* entity, is a relatively familiar kind of entity, while plural objects seem a little outré.

¹⁸ This is a claim that Russell later dropped, as his ontological commitment to classes lapsed.

¹⁹ Cf. *PoM*: §70: a class-as-many is 'the very kind of object of which *many* is to be asserted'.

²⁰ Compare the discussion of (Quod), §3.1.2 below.

many, Russell is faced with another ‘grave logical problem’. Given that \in is a two-place relation and α^M (I continue to use the suspect notation for convenience) is many, not only can the class-as-many not occur to the right of \in (this is the upshot of the first complication above), it cannot occur to the left of it either.

It seems now that the three options considered above face us once more: when ‘ α^M ’ occurs to the left of ‘ \in ’ it might represent a class-concept, the class-as-one, or ‘some other allied single entity’. Above I suggested that our best option (the best of a bad bunch) was to take the expression ‘ α^M ’ as representing a class-concept. In this case however, the correct course is that suggested by Landini (1998b: 71), who sees this issue as motivating the introduction of the class-as-one. Thus in, say,

$$(4) \quad \alpha^M \in \alpha^M$$

the first occurrence of ‘ α^M ’ should be understood as representing α^1 , the second as representing a class-concept.

But if this is correct, then α^1 has a very curious status. Its introduction is premised on the idea that it might explain how one class can be a member of another: that is to say, α ’s being a member of β is possible because although, in its primary sense, α is a class-as-many, there is yet some other sense by which an entity α^1 can go proxy for α . Now since Russell clearly also wants it to be distinct from the associated class-concept (*PoM*: §69), α^1 is left in a kind of limbo. It is not, at least not by intention, an intensional entity. But it is not either a class *in the primary sense*. What then is it? All we can say is that it is *that entity standing in relation C to the given class-as-many*; or as Russell puts it: ‘the class as one may be identified with the whole composed of the terms of the class’ (*PoM*: §74, 76).

The class-as-one is introduced, so it seems, on instrumental grounds, in order to satisfy Russell’s metaphysical demand that whatever *is* is a term, a logical subject. But it can only help in this regard if the fact that α^1 may be the logical subject of a proposition in any way tempers the fact that α^M cannot. But α^1 ’s being the logical subject of some proposition simply *cannot* temper the fact that α^M cannot be a logical

subject, for there is an ‘ultimate distinction’ between the class-as-one and the class-as-many, between, that is, α^1 and α^M . And in any case, Russell’s official response to ‘the contradiction always to be feared’ is *not* that the logical subject of such a proposition as that expressed by (5) is some related class-as-one (i.e. α^1).

$$(5) \quad \alpha^M \text{ is } \psi$$

Rather, it is to point out that ‘assertions can be made about classes-as-many, but the subject of such assertions is many, not one only as in other assertions (*PoM*: §127, 132). Thus the proposition expressed by (5) is not about α^1 , but the various members of α^M considered ‘together but severally’²¹. The introduction of classes-as-one does not, then, help to explain how it is possible for one class to be a member of another. It could only help if the class-as-one were *identical* to its associated class-as-many, and this, as I have stressed, is not the case.

The correct diagnosis of the problem here is, I suspect, that Russell was simply confused as to the status of the class-as-one. Given that he is seemingly unaware of the tension between his statement of the mixed paradox and his claim that the class-as-one and class-as-many are distinct, it is not implausible to suggest that he had a tendency to think of the two kinds of class as more closely connected than he was officially entitled to. The class-as-one is thought of as both: (i) bearing some extremely close relation (if not *identity* then at least *extensional equivalence as described by (C)*) to the class-as-many; and (ii) offering an account of the possibility of one class being a member of another. This is to attribute to Russell some serious confusion, but as we have noted above, he himself acknowledged a good degree of uncertainty on such matters.

The matters recently discussed are of necessity somewhat speculative, since my contention is that Russell had missed the tension at their heart. Nonetheless, we should, I think, accept that the mixed paradox of classes was not wholly extensional, but involved intensional aspects also. If this was not clear to his mind, but only confusedly present, then so be it. But that there is some intricate interplay of the extensional and intensional standpoints in Russell’s approach to the paradox of classes should come as

²¹ For the explanation of this idea see Chapter One, §4.2.2 and cf. *PoM*: §74.

no real surprise. Russell is adamant in *PoM* that the standpoints of *pure* extension and *pure* intension are the harsh extremes between which lie more fertile mixed positions (*PoM*: §66).²² Further evidence of the importance of the mixed paradox, for all its complications, is provided by the fact that even in the spring of 1906, when Russell was developing the ‘substitutional theory’, he saw one of its chief merits as being its resolving the problem of the class-as-one and the class-as-many (by not positing classes at all):

Of the philosophical consequences of the [substitutional] theory I will say nothing, beyond pointing out that it affords what at least seems to be a complete solution of all the hoary difficulties about the one and the many; for, while allowing that there are many entities, it adheres with drastic pedantry to the old maxim that, ‘whatever is, is one’. (*STCR*: 189)

3. Responding to the Paradoxes

We now turn to Russell’s strategy in the search for a solution to the paradoxes. I provide a brief overview of the various kinds of theory that he considered; however the main focus is not on the theories themselves, but on their relation to the theory of denoting. Two main issues arise in this regard: the matter of empty denoting phrases, and the relation between *meaning* and *denotation*. Empty denoting phrases are discussed in this section, the relation between *meaning* and *denotation* in §4.

Russell considers three kinds of response to the paradoxes: *type theory*, *restrictive theories* (as I call them), and *no-class theories*.²³ Of these, a simple theory of types is presented in *PoM* (§§497-500); the restrictive theories (e.g. the ‘zigzag’ and ‘limitation of size’ theories) are developed in the various manuscripts from 1903-05 and presented in the 1906 paper *Difficulties*; the no-classes theories (the ‘functional’ and ‘substitutional’ theories), as well as being considered in the 1903-05 manuscripts, are discussed in *Difficulties* and 1906’s *STCR*.

²² For instance it was, he says, the tendency to regard classes from the purely extensional standpoint which obstructed the correct logical theory of infinity (*PoM*: §131, 135).

²³ These classifications are not intended to be absolute. Type theories and no-class theories might both be considered restrictive in my sense; and no-class theories are not incompatible with type theories (witness *PM*). Moreover, as Urquhart (1988) notes, Russell did not always clearly distinguish his various theories: aspects of one theory often find their way into another.

3.1. The simple theory of types

In *PoM* Russell proposed, though without much confidence, a version of the simple theory of types (*PoM*: §§104, 497-500). The universe of terms is stratified into a hierarchy of logical types. A type is thought of as the ‘range of significance’ of a variable, that is, the range of terms which may be substituted for the variable in a propositional function to yield a proposition (*PoM*: §497). At the lowest level of the hierarchy, level 0, are individual terms (or just ‘individuals’). These form the range of significance of propositional functions of type 1, the next level of the hierarchy. Propositional functions of type 1 form the range of significance of propositional functions of type 2, i.e. those that take propositional functions of type 1 as argument. And so the hierarchy extends, in this manner, with the fundamental idea being that a propositional function of type $n + 1$ may only take arguments of type n . We have here a hierarchy of propositional functions; but, making the analogous moves, we could have formed a hierarchy of classes such that a class of type $n + 1$ can only have members of type n .

The hierarchy of types introduces restrictions on what can meaningfully be said. There will be expressions which *look* significant and well-formed, but which, when properly expressed, with suitable sensitivity to variations of type, turn out to be, strictly speaking, nonsense. Among these will be the problematic expressions associated with the paradoxes. Thus the paradox of classes becomes (using superscripts to indicate types):

$$(6) \quad w^n \in w^n \leftrightarrow w^n \notin w^n$$

But (6) is ill-formed: w^n can only have members of type $n - 1$, hence it can neither be nor not be a member of itself.²⁴

For technical reasons (including a failure to handle a paradox of propositions²⁵) which we need not examine here, the simple theory of types presented in *PoM* is

²⁴ Similarly for the paradox of predicates: ‘ $\phi^n(\phi^n) \leftrightarrow \neg\phi^n(\phi^n)$ ’ is nonsensical.

²⁵ *PoM*: §498ff.

inadequate; but a simple theory of types can be made to work.²⁶ And of course Russell himself would, eventually, come to endorse a theory of types (though a ramified, rather than simple, version²⁷). Why then is the theory only very tentatively put forward in *PoM*, and why is it not pursued in the unpublished manuscripts of 1903-05?

3.1.2. *Logical common sense: quodlibet ens est unum*

Russell identified three requirements of any solution of the paradoxes (*MPD*: 79-80).

The first, naturally enough, was that a solution should genuinely solve all of the paradoxes. The second was that the collateral damage to mathematics should be minimal, that the solution should not compromise any significant part of mathematics or any essential mathematical technique.²⁸ The third, and for our purposes most interesting, was that:

the solution should, on reflection, appeal to what may be called ‘logical common sense’—i.e. that it should seem, in the end, just what one ought to have expected all along. (*MPD*: 79-80)

(Russell illustrates with a criticism of Quine, whose system, though flush with ingenuity and ‘logical dexterity’, seems to be ‘created *ad hoc* and not to be such as even the cleverest logician would have thought of if he had not known of the contradictions’ (*MPD*: 80).)

The simple theory of types given in *PoM* fails, in the main²⁹, for want of accord with logical common sense. To introduce the hierarchy of types is to enforce restrictions upon the variables of logic; but for Russell there seemed to be no reason whatsoever, independently of the paradoxes, to make such a restriction. Thus while the theory of types may be a technical fix, it lacks *philosophical justification*. It is the need for philosophical justification to which the third requirement quoted above appeals.

²⁶ For the differences between Russell’s *PoM* version and later more successful formulations see, e.g., Urquhart 1988: 82-3.

²⁷ See *ML*, *PM*.

²⁸ Cf. Russell’s concern for mathematical induction in *Insolubilia*. See Stevens 2005: 58-61 for discussion.

²⁹ I will ignore its technical shortcomings, since these are resolvable.

Russell is committed to the *doctrine of the unrestricted variable*. This is an extremely important theme in his work, and it is as well to clarify, at this point, quite what it commits him to. In so doing we will reveal its connection with ‘logical common sense’ and the unease that Russell feels with regard to the theory of types.

The doctrine of the unrestricted variable is naturally thought to embody a commitment to the view that the variable ranges over absolutely everything that is (has being). This however, is not a perfect fit with Russell’s position in *PoM*, for reasons that will become apparent. The doctrine should rather be understood (at least in *PoM*) as committing one to the view that, even though there are many kinds of entities (things, concepts, propositions, etc.), there is only one kind of genuine variable: the entity variable. Let’s isolate that claim:

(UV) *Doctrine of the unrestricted variable*: there is only one kind of variable, the entity variable.

Unfortunately, there is a tendency in the literature to conflate (UV), to which Russell was, I agree, committed, with the doctrine *quodlibet ens est unum*.

(Quod) *Quodlibet ens est unum*: whatever is, is one.

These two theses have been put to work in Russell scholarship in the laudable attempt to demonstrate a good deal more unity to Russell’s work than he is sometimes credited with.³⁰ They ought, nonetheless, to be kept apart, for the very good reason that Russell is committed to (UV) but denies (Quod) (at least in *PoM*).^{31 32}

That the two claims are run together is evident in two recent important works: Graham Stevens’s *The Russellian Origins of Analytic Philosophy* (2005) and Gregory

³⁰ The view that Russell’s work lacks unity is summed up in C. D. Broad’s famous quip that Russell produced ‘a new system of philosophy every few years’ (Broad 1924: 79). For a recent attempt to demonstrate its falsity see Stevens 2005; for a less recent attempt see Weitz 1951.

³¹ In *OnF* (written 1904) and sections of *OF* (written 1905), Russell acknowledges ‘mode of combination’ and ‘meaning’ variables. I do not see in this reason to suppose that Russell had abandoned (UV). It would be foolish to place too great an emphasis on unpublished material that is not reflected in any contemporary (to it rather than us) published material.

³² It should be borne in mind that Russell himself was not particularly careful in keeping the two views distinct. I thank Graham Stevens for drawing my attention to this point.

Landini's *Russell's Hidden Substitutional Theory* (1998a). Stevens describes (UV) as 'arguably the most important doctrine espoused by Russell in [*PoM*]' (2005: 6), and continues:

As a formal doctrine, [(UV)] amounts to the requirement placed on any calculus of logic that it should have only one kind of variable (ranging over everything in the universe). The philosophical insight that underlies this doctrine is Russell's belief that there are no distinctions in type between existing things. Everything, that is, is of the same logical type, according to Russell. As he liked to put it, 'whatever is, is one' [*STCR*: 189]. (Stevens 2005: 6)

Similarly, Landini argues that (Quod) dominated Russell's thinking in the period from the publication of *PoM* to the publication of *PM*, and suggests that, for Russell, the thesis amounted to the claim that:

any calculus for pure logic must treat all entities alike; it must adopt only one style of pure and unrestricted "entity" variable *which regards as one whatsoever is*. (Landini 1998a: 3, emphasis added)

Both commentators, then, run (UV) and (Quod) together.³³

However, contrary to Landini's claim that 'Russell's allegiance to the doctrine [(Quod)] never wavered' (1998a: 4), it may be easily established that at some stages at least, he explicitly *denied* it, as in the following passage:

Whatever is, is one: being and one, as Leibniz remarks, are convertible terms. It is difficult to be sure how far such statements are merely grammatical. For although whatever is, is one, yet it is equally true that whatever are, are many. But the truth seems to be that the kind of object which is a class, *i.e.* the kind of object denoted by *all men*, or by a concept of a class, is not *one* except where the class has only one term, and must not be made a single logical subject. There is [. . .] in simple cases an associated single term which is the class as a whole [*i.e.* as-one]; but this is sometimes absent, and is in any case not identical with the class as many. (*PoM*: §127, 132)

³³ Stevens is on surer ground here than Landini here. He, in effect, casts (Quod) as the philosophical underpinning of (UV), thereby distinguishing them. It remains the case, however, as argued below, that the Russell of *PoM* did not endorse (Quod).

Landini's claim is not, I take it, merely the grammatical point to which Russell refers.³⁴ It is of course true that whatever is—in *the singular*—is *one*. But Russell is adamant that there are *also* such objects as classes-as-many. And while these cannot be said to *be* (in the singular), nonetheless they *are*, they have a significant and positive ontological status. We might put the point in this way: enumerating everything that *is* (in the singular) would not, according to Russell, exhaust the inventory of the world, for it would miss out the classes-as-many. This seems to capture Russell's position as stated above. But Landini's understanding seems to be that such an enumeration *would* exhaust the inventory of the world.

Stevens, as quoted above (2005: 6), renders (Quod) as tantamount to (or at least the philosophical justification of) the claim that everything is of the same logical type; but it is unclear why (Quod) should embody any commitment to a view of types. We should distinguish two ways in which a purported entity may fail to be a possible value of the variable. One is that it is of the wrong logical type. The other is that it is not really an entity—i.e. term—but rather an object (and so irreducibly plural). (Quod) commits one to the view that whatever has positive ontological status is *one*: it does not commit one to the view that everything is of the same type.

Stevens continues (not using 'object' in Russell's technical sense, but rather as synonymous with 'entity'):

If something is an object, Russell thought, then it is an object pure and simple and ought to be amenable to the same treatment as any other object from the point of view of logic. [. . .] It ought to be the kind of thing that can have certain properties predicated of it, for example, or that can be quantified over; in short, *it should be possible to make it into a logical subject*. (Stevens 2005: 23, emphasis added)

Landini adopts the same position.

³⁴ I take it that this is not his position, though if it *were* it might explain his citing the passage from which I have recently quoted in support of the claim that (Quod) is the 'fundamental doctrine' of *PoM* (1998a: 54). Given that he is not making the cheap grammatical point, I fail to see how the passage supports his view.

the fundamental doctrine [of *PoM*] “whatever is, is one” can be otherwise expressed as “Whatever has being is a logical subject”, and even more revealingly as “Whatever has being can occur as a term [logical subject] of a proposition.” (Landini 1998a: 55)³⁵

This makes it a prerequisite of having ‘positive’ ontological status that an object be a logical subject of some proposition. But consider a given class-as-many (we’ll call it ‘*m*’³⁶): given that *m* is a class-as-many, it is not *one* but *many* (irreducibly plural) and, as such, it cannot be a logical subject.

In that case, we ought to have a contradiction: for here we have an object *m* which is not a possible logical subject. But, as we noted in Chapter One (§4.2.2), the threatened contradiction is ‘solved’ by denying that it arises; and that denial is motivated by an appeal to the distinction between terms and objects (i.e. that *m* is not a term, but an object). So the fact that the object *m* cannot be a logical subject in the same sense as a term can, does not, *contra* the accounts of Stevens and Landini, disqualify it from having ‘positive’ ontological status. It simply does not follow that whatever is, is one.³⁷

We should, moreover, be careful to distinguish two senses in which ‘one-ness’ might be ascribed to an entity. Russell is careful to point out that there is a distinction between (as we might put it) *being one in the sense of being a unified (or simple) term*, and (as we might put it) *numbering one*:

³⁵ Cf. Landini 1992: 170-71.

³⁶ As we saw in §2.1 above, introducing such an expression as ‘*m*’ is not strictly legitimate; I do so for the purposes of bringing out the point in simple terms.

³⁷ Further evidence against Landini’s interpretation stems from his, as I see it, suspect reading of the theory of denoting concepts. Discussing the strange ‘combinations’ of entities—i.e. objects—he writes:

What, after all, are “combinations of terms”? Is there a single logical subject ‘Brown or Jones’? Russell’s official answer should be “no.” But at times he seems to demur in spite of the obvious threat to the fundamental doctrine of *Principles* [footnote to *PoM*: §58, 55n, in which the notion of *object* is first introduced]. In the end the most charitable interpretation is to take Russell’s use of “combinations of terms” simply as a heuristic device to help in clarifying the kinds of denoting concepts. (Landini 1998a: 60)

But in fact the most charitable interpretation is that Russell meant what he said—objects are irreducibly plural—and that Landini’s having taken (Quod) to be the fundamental doctrine of *PoM* is wide of the mark. Dau (1986), in his excellent study, distinguishes two conflicting versions of the theory of denoting concepts at play in *PoM*, and highlights the fact that the class-as-many is the only kind of object posited by both versions. This strongly suggests, it seems to me, that the class-as-many was more than just a heuristic device.

As regards the fact that any individual or term is in some sense *one*, this is of course undeniable. But it does not follow that the notion of *one* is presupposed when individuals are spoken of: it may be, on the contrary, that the notion of term or individual is the fundamental one, from which that of *one* is derived. (*PoM*: §125, 130)

That every term is *one* is a *metaphysical* fact about terms: simple terms are *units*; complex terms have a certain kind of *unity*. But *being a unit* is not, fundamentally, an *arithmetical* property. The only kind of thing that can be *one* in the arithmetical sense is a class, namely a class-as-many having only one member. The attenuated sense in which any *term* is arithmetically one is merely that it may be taken as forming a unit class, the cardinality of which is one.³⁸ But Russell's position is that it is essential for logicism that a unit class and its member not be identified.

Stevens and Landini might respond to all of this by claiming that the class-as-many is nothing over and above its members.³⁹ This would be, in effect, to charge my position with having 'over-ontologised' the class-as-many. But this cannot be right. The class-as-many is more than just the many entities that compose it: it has a certain kind of unity that distinguishes it from the mere aggregation of its terms.⁴⁰ Thus the members of a class-as-many have enough unity to make them *many*, rather than simply *disparate*, but not enough to make them *one* (cf. Chapter One, §4.2.2).

Moreover Landini's claim that the intensional viewpoint is primary overlooks the complex interplay of intension and extension discussed in §2. This complex interplay was no passing fancy: it continues into Russell's post-*PoM* manuscripts. I quote at length:

The fact revealed by the Contradiction is this: there are collections of terms which can be defined *distributively*, in the sense that, given any term whatever, we can decide whether or not it belongs to the collection, but which cannot be defined *collectively*, *i.e.* cannot be defined as all terms having such and such a property, or as all the members of such and such a class. If this is the right view, it elicits very clearly the necessity of taking account of intension as well as

³⁸ Cf. *PoM*: §128, 132.

³⁹ Cf. Landini 1998a: 70-71; Landini 1992: 164.

⁴⁰ *PoM*: §70, 69.

extension in dealing with classes. In fact, a class is *an extension defined by an intension*; we cannot work classes except by taking them in extension, and yet it is essential that every class should be defined by an intension. (FN: 157)

I have argued that (UV) and (Quod) come apart, and that Russell was, in *PoM*, committed only to the former. (UV) is given only as a claim about what kind of variables there are, not what their range is. The claim that I have sought to deny is that the variable ranges over absolutely everything that has being: it does not, I claim, range over classes-as-many. Now if the variable is on the one hand ‘unrestricted’ and on the other does not include classes-as-many in its domain, is my position not embroiled in a contradiction?

There are three points to make in response to this worry. The first is that as we have seen (§2), there is good reason to suppose that Russell was simply confused as to the relation between classes-as-many and their associated classes-as-one. Russell took it, I think (and I take it that I am agreeing with Landini here), that truths about classes-as-many were somehow expressible in terms of truths about their associated classes-as-one. This is to attribute to Russell a confusion, but so be it. The second, related to the first, is that even if a class-as-many is not a possible value of a variable, it does not follow that it is beyond our cognitive reach. Among the values of variables are all the denoting concepts that denote classes-as-many. So while there may not be any possibility of *direct* knowledge of classes-as-many, the possibility of denotative knowledge remains. The third is that although a class-as-many is not a possible value of the variable, it does not ‘stand outside logic’ (in Wahl’s (1993: 74) phrase) in at least the following respect: every class-as-many is, *in a certain sense*, the logical subject of some proposition:

In such a proposition as “*A* and *B* are two,” there is no [single] logical subject: the assertion is not about *A*, nor about *B*, nor about the whole composed of both. But strictly and only about *A* and *B*. Thus it would seem that assertions are not necessarily *about* single subjects, but may be about many subjects. (*PoM*: §74, 76-7)

That is, *A-and-B*—a class-as-many—is not a *single* logical subject, but is not either outside logic since it can occur in propositions having many subjects.

It is important to be clear as to the distinction between (UV) and (Quod) because otherwise one might misconstrue Russell's concerns about the simple theory of types. Type-restricted variables offend against 'logical common sense' not because they are restrictive *simpliciter*, but because the restrictions themselves are not such as would seem obvious to any logician unaware of the paradoxes. On my view, the variables of *PoM* are, in one sense, *already* restricted: they range only over what is *one* (not also over what are *many*). Now this might seem to offend common sense, but Russell is able to convince himself (with some effort, no doubt) that it does not offend *logical* common sense, for logical common sense is sensitive to the requirements of logical theory, and in particular the nature of classes. That is to say, given that classes-as-many are irreducibly plural—and remember that a class is, in its primary sense, a class-as-many—it simply follows, as a matter of logical common sense, that they are not *single* logical subjects. The problem with the theory of types is that its hierarchy is not imposed on the basis of some *positive* insight into the nature of things. In Russell's manuscripts from 1903-05 we increasingly find appeals to 'direct inspection'. Direct inspection into the nature of classes reveals (let us suppose with the Russell of *PoM*) that classes-as-many are irreducibly plural: direct inspection into the nature of the universe does not reveal it to be stratified into a hierarchy of types.⁴¹ Thus the imposition of such a hierarchy requires a strong philosophical justification. This is precisely what the theory of types, at least in 1903, lacked.

Overtuning the Landini-Stevens interpretation enables us to recognise, what might otherwise remain obscure, a respect in which the theory of descriptions—which spurred the development of the substitutional theory—represents an advance from the *PoM* position. In Russell's post-*PoM* work, and certainly by the time he began to develop the substitutional theory, he is generally taken to endorse both (UV) and (Quod). The substitutional theory, insofar as it abandons any commitment to classes, may now be seen as allowing for the adoption of both principles. This is the respect in which it 'affords what at least seems to be a complete solution of all the hoary difficulties about the one and the many' (*STCR*: 189). Russell records, with satisfaction, that the

⁴¹ At least such was Russell's position in 1903. Agreement between parties as to what direct inspection reveals is obviously not assured, nor even is agreement between one's earlier and later selves (as Russell's example shows).

substitutional theory ‘adheres with drastic pedantry to the old maxim that, “whatever is, is one”’ (*STCR*: 189). The satisfaction must have been all the sweeter for the fact that in his earlier work, with his commitment to classes-as-many, he had not been able to accommodate the maxim at all. Direct inspection and logical common sense now reveal that whatever is, is one. Common sense had, I suspect, revealed this all along⁴²; but the exigencies of the *PoM* theory of classes—in particular the primacy of the class-as-many, relative to the class-as-one—determined that the Russell of *PoM* saw *logical* common sense as revealing something else.

3.2. ‘Restrictive’ and ‘no-classes’ approaches

Not content with the prospects of type theory, Russell’s energies focused on two other forms of theory, one in which class-formation is restricted in *some* cases, and one in which class-formation is restricted in *all* cases. Under the head of the former are included his ‘zigzag’ and ‘limitation of size’ theories; the latter includes the functional theory of 1903 and the ‘substitutional theory’.

3.2.1. Restrictive theories

The zigzag theory builds on a line of thought that had begun in *PoM*. There, Russell had identified as a common factor in the various formulations of the paradox a certain kind of dependence among variables:

in the type of propositional functions we are considering [. . .], the argument is itself a function of the propositional function: instead of ϕx , we have $\phi\{f(\phi)\}$, where $f(\phi)$ is defined as a function of ϕ . Thus when ϕ is varied, the argument of which ϕ is asserted is varied too. Thus “ x is an x ” is equivalent to: “ ϕ can be asserted of the class of terms satisfying ϕ ,” this class of terms being x . If here ϕ is varied, the argument is varied at the same time in a manner dependent upon the variation of ϕ . For this reason, $\phi\{f(\phi)\}$, though it is a definite proposition when x is assigned, is not a propositional function, in the ordinary sense, when x is variable. Propositional functions of this doubtful type may be called *quadratic forms*. (*PoM*: §103, 104)

His thought was that some functions of the quadratic form will be held to determine a class-as-many, but not a class-as-one.⁴³ The zigzag theory starts from ‘the suggestion

⁴² This would go some way towards explaining Russell’s failure to clearly distinguish (UV) from (Quod).

⁴³ Cf. *PoM*: §§101, 104, 127, 484.

that propositional functions determine classes when they are fairly simple, and only fail to do so when they are complicated and recondite' (*Difficulties*: 145-46). The kind of complication in question is that which gives rise to quadratic forms, effectively a kind of self-reference.⁴⁴

Two main problems attend such a theory, Russell claims (*Difficulties*: 147). Firstly, the technical details of the theory turn out to be extremely complicated. Secondly, and this is by now a familiar theme, the requisite restrictions on class-formation lack adequate philosophical justification: 'I have found no guiding principle except the avoidance of contradictions', Russell admits (*Difficulties*: 147).⁴⁵

The 'limitation of size' theory cautions against the formation of classes from propositional functions involving 'self-reproductive processes' (*Difficulties*: 152). The process of collecting the class of *all terms* is, for example, self-reproductive in that it generates a *new* term—the class of all terms. Collecting this class (this new term) together with its members generates a further class (a further new term), and so on. On this kind of basis⁴⁶ self-reproducing classes are excluded, with the consequence that there is, for example, no universal class (consisting of absolutely everything). The difficulty with this kind of theory, Russell says, is that 'it is not easy to see how to state such a limitation precisely' (*Difficulties*: 154).⁴⁷ And, once more, logical common sense is bypassed: independently of Cantorian paradoxes, for instance, there seems no justification for the exclusion of the universal class.

The zigzag and limitation of size theories both propose some restriction upon the formation of classes from propositional functions. Of course, then, the nature of propositional functions and their relation to classes is of the utmost importance to

⁴⁴ For plausible explanations of the sense in which quadratic forms are 'zigzaggy' see Bostock 2009: 120-121 or Urquhart 1988: 85.

⁴⁵ Bostock (2009: 121) objects on different grounds. In general, even if a given class is definable in terms of a problematic function, it will also be definable in terms of unproblematic functions. This is a general worry for all restrictive theories.

⁴⁶ This is a gross simplification. To be precise would require discussion of the Burali-Forti paradox and the theory of ordinals.

⁴⁷ Again, Bostock (2009: 123) offers an objection to the general approach. Even if it were successfully formulated for the paradox of classes, it is unclear how it could be extended to (e.g.) the paradox of predicates.

Russell in this period. Before discussing this relation in more detail, I will introduce the no-classes theories.

3.2.2. No-classes theories

The ‘functional theory’, which Russell pursued in 1903 after the publication of *PoM*, involved the elimination of all classes and the implementation of a system of functions and arguments in something like the manner of Frege.⁴⁸ Briefly Russell thought he had solved the paradoxes in this way (Grattan-Guinness 1977: 78). However, the emergence of a paradox of non-self-applicable functions (obviously analogous to the paradox of predicates) soon scuppered this line of thought.

The second attempt at a ‘no-classes’ theory—the substitutional theory—yielded far greater success than any of Russell’s previous approaches, but only once the theory of descriptions was up and running.⁴⁹ The theory requires neither classes nor propositional functions. Instead of propositional functions we have *matrices* (written ‘ p/a ’) and *substitutions* (written ‘ x/a ’). The symbol

$$\bullet \quad p(x/a)!q \qquad \text{also written:} \quad p/a!x!q$$

is understood to mean: ‘ q results from p by substituting x for a in all those places (if any) where a occurs in p ’ (*STCR*: 168). The symbol ‘ p/a ’ is treated as an incomplete symbol which we might think of as abbreviating the phrase ‘the result of replacing a in p by ___’.⁵⁰ Instead of speaking of classes and class-membership, we instead speak of substitutions yielding truths:

To say that x is a member of the class α is now to say that for some values of p and a , α is the matrix p/a and $p/a!x$ is true. (*STCR*: 172)

Matrices such as p/a in this way do duty for the classes that are no longer admitted. (It should be noted that Russell need not *deny* that there are such things as classes, but can simply ‘bracket’ the question. This appears to be his position in *Difficulties* (154),

⁴⁸ See Urquhart’s introduction to *CP4* (xx-xxiii) for further details.

⁴⁹ See: Hylton 1980; Weiss 1994; Landini 1998a; 2003; and Stevens 2005.

⁵⁰ Incomplete symbols are discussed in Chapter Three.

though in *STCR* (166) he is more forthright ('there are really no such things as classes'⁵¹); by *PM* (72) his agnosticism has returned.)

On this theory the kinds of predications that generate the paradoxes are outlawed by the grammar of substitution:

now ' x is an x ' becomes meaningless, because ' x is an α ' requires that α should be of the form p/a , and thus not an entity at all. (*STCR*: 172)

That is to say, the nearest we can come to formulating the paradox is to put: ' $p/a \cdot p/a$ '. This is roughly equivalent to the nonsensically incomplete 'the result of replacing a in p by the result of replacing a in p by $__$ '. In this way the restrictions on what can be significantly formulated—which had appeared *ad hoc* on the theory of types—simply fall out of the grammar of the substitutional theory.⁵²

The above is a very inadequate outline of the substitutional theory. (I shall discuss it again in Chapter Five.) It suffices, however, for our purposes here, as it serves to indicate the great importance of the theory of denoting. Matrices are treated as incomplete symbols, having no significance in isolation from their proper contexts. This is the key to the application of the substitutional theory, and underlies Russell's claim that his pre-*OD* attempts at a substitutional theory had 'failed for want of the theory of denoting', but that 'as a consequence of the new theory of denoting, I found at last that substitution would work, and all went swimmingly' (Grattan-Guinness 1977: 80).^{53 54}

3.3. *Empty denoting phrases*

We must beware of falling into step with a prevalent trend in the literature, namely, acknowledging the importance of the theory of descriptions in the development of the substitutional theory, and then neglecting to say anything more to the connection of the theory of denoting and the paradoxes.⁵⁵ It is not enough, for a proper understanding of

⁵¹ But cf. *STCR*: 188.

⁵² See e.g. Stevens 2005: ch. 2 for discussion.

⁵³ In Chapter Five I discuss the differences between the theory of descriptions and the earlier theory with the aim of ascertaining why the former, but not the latter, could support the substitutional theory.

⁵⁴ The substitutional theory ultimately succumbed to a paradox of propositions, but I will not discuss this. See Landini 1989; Stevens 2005: ch. 3.

⁵⁵ See, for example: Chihara 1973: 14-15, 18n.; Farrell Smith 2005: 161; Landini 2003; Stevens 2005: 46ff; 2009: 30-31. Of course the focus of such authors may be different to my own.

Russell's work in the period 1903-05, to only draw the connection between the *theory of descriptions* and the paradoxes: for the most part, Russell's work in this period was conducted prior to the discovery, and hence in ignorance, of the theory of descriptions. Moreover, the majority of Russell's work from 1903-05 centred not on the substitutional theory but the zigzag theory. Thus we must speak more to the connection of denoting to the restrictive theories.

Some commentators emphasise the connections between the paradoxes and the question of empty denoting phrases. To take just one example⁵⁶, Michael Potter writes:

Since it was [. . .] evident to Russell that the solution [of the paradoxes] would involve accepting that some phrases which apparently denote classes do not in fact do so, he took an especial interest in cases (such as 'the present king of France') where the denoting concept does not denote anything. (Potter 2004: 123)

This approach is along the right lines, but there is some detail to be filled in.

In general, we have seen above, Russell's approach is to introduce a restriction of some kind upon the formation of classes-as-one from propositional functions. That is, while every propositional function determines a class-as-many, not all also determine a class-as-one. This must, as a matter of course, imply a correlative adjustment to the theory of denoting. Let the propositional function in question be $\phi\hat{x}$. The claim is, then, that $\phi\hat{x}$ determines a class-as-many, α^M , but no class-as-one, α^1 . Following Russell's proposal (*PoM*: §70) that there is only a corresponding predicate to those propositional functions that determine classes-as-one, we may assume that there is no predicate ϕ corresponding to $\phi\hat{x}$.⁵⁷

In Chapter One we saw how denoting concepts are derived from predicates (class-concepts) by a kind of 'logical genesis' (*PoM*: §57). But we now have a case where a denoting concept is derived from a propositional function for which there is no

⁵⁶ See also Kilmister 1981: 101; Levine 2005.

⁵⁷ He suggests, for example, that 'the ϕ in $[\phi\hat{x}]$ is not a separate and distinguishable entity: it lives in the propositions of the form $[\phi\hat{x}]$, and cannot survive analysis' (*PoM*: §85, 88).

correlative predicate.⁵⁸ The class-as-many α^M is determined by $\phi\hat{x}$ (not from ϕ , *ex hypothesi*), and yet α^M is a ‘numerical conjunction’, i.e. the kind of object denoted by a denoting concept of the form $\langle \text{all } \phi \rangle$ (which appears to contain, or at least be closely related to, the predicate ϕ). This is a bit of a tangle, which is to be unravelled only by a complete and clear explication of the concepts of (at least) *assertion*, *class-concept*, *denoting concept*, *predicate*, and *propositional function*. Certainly no such explication is forthcoming in *PoM*, nor in the unpublished papers in *CP4*. Russell draws many fine distinctions in this area and begs his reader at the beginning of his discussion of classes ‘not to regard as idle pedantry the apparatus of somewhat subtle discriminations to be found in what follows’ (*PoM*: §66, 66). In the end, however, so many distinctions are introduced that the position is of only questionable coherence. The distinction between *class-concept* and *predicate* is a case in point. In at least three places, Russell avers uncertainty as to the strict metaphysical distinction between the class-concept and predicate. The most striking of these I quote at length:

It must be held, I think, that every propositional function which is not null defines a class, which is denoted by “ x ’s such that ϕx .” There is thus always a concept of the class, and the class-concept corresponding will be the singular, “ x such that ϕx .” But it may be doubted—indeed the contradiction [. . .] gives reason for doubting—whether there is always a defining predicate of such classes. Apart from the contradiction in question, this point might appear to be merely verbal: “being an x such that ϕx ,” it might be said, may always be taken to be a predicate. But in view of our contradiction, all remarks on this subject must be viewed with caution. (*PoM*: §84, 88)⁵⁹

Notice the phrase: ‘Apart from the contradiction in question, this point might appear to be merely verbal’. Indeed the point *does* appear merely verbal. Where we have a class collected by a propositional function $\phi\hat{x}$, why may we not take that class to be collected by the predicate ϕ ?⁶⁰ That seems, in fact, to be precisely what Russell *allows* in positing the class-concept ‘ x such that ϕx ’. Independently of the paradoxes, there seems no good answer to this question (and let us not forget Russell’s appeals to ‘logical common sense’). Moreover, the view that predicates and propositional functions come and go together is supported by Russell’s post-*PoM* manuscripts: ‘Now a property, in its most

⁵⁸ That propositional functions (as opposed, or in addition, to predicates) may give rise to denoting concepts is an interpretation shared by Klement (2009: 69-70).

⁵⁹ Cf. *PoM*: §57, §58.

⁶⁰ The mode of collecting may be different in the two cases, but that does not appear to be problematic.

general form, is a propositional function' (*Classes*: 5); 'we shall speak of the sine, the logarithm, the square, being a man, the property that if one is a man one is a mortal, as functions' (*Functions*: 51). In *PoM*, properties are naturally construed as predicates; so if predicates are properties, and properties in their 'most general form' are propositional functions, then predicates and propositional functions would seem to come and go together.

If, as seems plausible, there is really no principled objection to the correlation, in all cases, of $\phi\hat{x}$ with ϕ , then we have on our hands such denoting concepts as <the class-as-one of ϕ s>, a denoting concept which does not denote anything (there being, in our example, no α^1 for it to denote). It seems extremely likely that this is the source of Russell's concern, so prominent in *OD*, with 'empty' descriptions.

This general account is supported by consideration of Russell's unpublished papers from 1903-05. One may still, however, find it surprising that in *OD* the question of empty denoting phrases is couched in terms that seem so entirely unrelated to the paradoxes. The explanation is partly that Russell was addressing the readership of *Mind*, not all of whom could be expected to be familiar with the foundations of mathematical logic. I suspect, moreover, that Russell felt no unease in presenting his position in a more 'popular' style since this reflected the wider⁶¹ philosophical aspects of the theory of denoting. In *OD* Russell wrote that 'The subject of denoting is of very great importance, not only in logic and mathematics, but also in theory of knowledge' (*OD*: 41). Given the close connection of logic to metaphysics in Russell's thought, this is effectively to claim that denoting is of very great importance to both metaphysics and epistemology, which is to make it one of the central issues in the western tradition. Its importance is, then, in some sense independent of the paradoxes, though of course since the paradoxes undermine a certain logical—and so for Russell *metaphysical*—innocence, they are hardly less central than the question of denoting.

The wider philosophical concerns of the theory of denoting, at least as discussed in Russell's 1903-05 manuscripts, include particularly the question of 'objective

⁶¹ That is, wider than mathematical logic strictly construed.

falsehoods'.⁶² This was a problem arising from the combination of Russell's theory of propositions with his account of truth. I shall briefly discuss this below (§4.1), when the theory of meaning and denotation has been introduced. Beforehand, and in part to motivate the discussion of that theory, I pause to point out the reason why the question of empty denoting phrases cannot be the whole story of the relation between denoting and the paradoxes.

The 'Standard View' (as Griffin (1996) labels it) of the origins of the theory of descriptions has it that Russell's ontology in *PoM* was 'unrestrained' in some (usually pejorative) 'quasi-Meinongian' sense, and that the role of the theory of descriptions was to enable Russell to eliminate from it such shady characters as the present king of France, the round-square, and the golden mountain. This view is, however, almost certainly wrong and is almost universally rejected by Russell scholars (I shall discuss this in a little more detail in Chapter Four).

To one in the grip of the Standard View, empty descriptions will assume a lead role in the story of *OD* and the development of the theory of descriptions. Having rejected the Standard View, we are in a position to reemphasise the status of theories of denoting as accounts of generality. A theory of denoting must give not only an account of *what* certain propositions are about, but also an account of *how* they are about it. For example, a sentence containing a denoting phrase is about whatever the denoting concept indicated by that phrase denotes. Thus (7) is not about the entity occurring in subject-position in ⟨7⟩, but what it denotes, i.e. Socrates.

(7) The teacher of Plato is wise.

⟨7⟩ ⟨<the teacher of Plato> is wise⟩

But we also want our theory of denoting to provide an explanation of *why* (7) is about Socrates. For this reason, it does not suffice to stipulate that the proposition expressed by (8) is about, say, the null-class.⁶³

⁶² There is also a growing concern with epistemological issues.

⁶³ Or whatever other conventional denotation one chooses to posit. See Levine 2005 for an excellent discussion of Russell's objections to the imposition of conventional denotations.

- (8) The present king of France is wise.

As Russell points out in *OD*, the stipulation that (8) is about the null-class does not lead to logical error, but it ‘is plainly artificial’ (*OD*: 47). Direct inspection does not reveal that (8) is about the null-class; and in virtue of *what* is the null-class apt to be denoted by <the present king of France>? To say more, to sharpen the analysis, would require an account of the relation between <the present king of France> and the null-class. In general then, I think it is fair to say that Russell’s interest in empty denoting phrases is, at least partly, sparked by the need to give a genuinely explanatory account of the relation between that which denotes and that which is denoted—not merely by the difficulty of providing a denotation where one is apparently absent.

4. The Theory of Meaning and Denotation

I shall call the theory of denoting that Russell develops in the 1903-05 papers in *CP4*⁶⁴ ‘the theory of meaning and denotation’. It is largely the same as the theory of denoting concepts as set out in Chapter One. One might characterise the difference between the two in following terms.

Unlike ‘denoting concept’, the words ‘meaning’ and ‘denotation’ have both a substantive and a verbal sense. It is useful to think of the difference between the theory of denoting concepts and the theory of meaning and denotation as amounting to little more than a shift of emphasis, *from* the substantive sense of ‘meaning’ (in which it is synonymous with ‘denoting concept’) *to* the verbal sense. That is to say, whereas in *PoM* the focus had been primarily upon the denoting concepts and denoted objects (e.g. *PoM*: §61), Russell’s prime concern in formulating the theory of meaning and denotation is the *relation* of denoting, the relation between the *meaning* (i.e. denoting concept or, in Russell’s increasingly frequent term, denoting *complex*) and the *denotation*. Of course it is not the case that the relation of denoting is of no concern to Russell in *PoM* (very much the contrary), nor that the nature of meanings and denotations are of no concern to Russell in developing the later theory (again, very much the contrary). But it is useful to fix upon a difference between the two approaches since, firstly, the two theories are ultimately extremely similar⁶⁵, and secondly, the shift

⁶⁴ Especially those from §III of *CP4*: *OMDP*, *DVD*, *PAD*, *OMD*, and *OF*.

⁶⁵ Wahl (1993) agrees.

of emphasis serves to indicate the nature of Russell's application of the theory of denoting to the paradoxes. Much of what is to be said about the theory of meaning and denotation will therefore be familiar. Nonetheless it is useful to plot the development of Russell's position in order to get a feel not only for the shift of emphasis that occurs, but also for the kind of modifications he considered and rejected.⁶⁶

In the earlier of Russell's 1903-05 manuscripts a particularly significant modification is mooted, though it is ultimately not embraced. Here the *PoM* position is modified to incorporate *two* word-world relations: *meaning* and *denoting*⁶⁷. This indicates the growing influence on Russell, at this time, of Frege, and begins to give us an insight into Russell's claim in *OD* that his earlier theory had been 'very nearly the same as Frege's' (*OD*: 42n).⁶⁸ It will be noted below, however, that Russell's position in the early manuscripts is certainly not identical to Frege's, and the position arrived at in the manuscripts immediately preceding *OD* is far closer to the *PoM* view than to Frege's (and is only superficially similar to the latter⁶⁹).

On Russell's view in *OMDP* (written 1903), proper names have denotation but no meaning. Thus 'Arthur Balfour' is 'destitute of *meaning*, but *denotes* an individual' (*OMDP*: 284, 'denotes' is here used in its linguistic sense). Verbs and adjectives, on the other hand, have meaning, but do not denote (*OMDP*: 284). Thus far, though we have two linguistic relations—meaning and denoting—the view is otherwise unlike Frege's in that whereas for Frege all expressions *both* mean and denote (i.e. have *Sinn* and *Bedeutung*), for Russell names *only* denote, and verbs and adjectives *only* mean.

The entities that are meant by verbs and adjectives (relations and predicates) can, however, also be denoted (in the linguistic sense). The concept that is meant by the adjective in (9) is denoted by the substantive 'Blackness' in (10).

⁶⁶ The unpublished papers are discussed in some detail by: Rodriguez-Consuegra (1989; 1992); Wahl (1993); and Klement (2004a; 2004b). None of these entirely satisfy me (though the Klement papers are *very* good).

⁶⁷ Notice that here 'denoting' is used for a *linguistic* relation. Russell also uses it for the *logical* relation holding between a meaning (denoting concept) and its denotation. I will not provide a disambiguating terminology but will indicate which sense is intended as necessary.

⁶⁸ In Coffa's (1980: 57) terms, the Russell of *PoM* was a 'semantic monist'; at this stage he is flirting with 'semantic dualism'.

⁶⁹ As such it may be misleading to follow Klement (2004b: 16) in labelling 1903-05 Russell's 'Fregean Period'.

- (9) The table is black.
- (10) Blackness can make seeing ants difficult.

The similarity to Frege is closest with regard to denoting phrases. ‘The table’ in (9), has *both* meaning and denotation: it *means* a particular meaning (i.e. <the table>) and *denotes* a particular table⁷⁰. A difference from the theory of denoting concepts should be noted here. On the theory of denoting concepts the significance of a denoting phrase is, strictly, exhausted by its indicating a denoting concept. Thus that ‘the teacher of Plato’ indicates <the teacher of Plato> is a sufficient condition for its significance. The fact that <the teacher of Plato> denotes Socrates is interesting, but has no bearing on the ultimate significance of the related denoting phrase. Of course one *may*, if so disposed, posit an additional word-world relation, holding between ‘the teacher of Plato’ and Socrates. This would be denoting in a linguistic sense. But, crucially, this relation—unlike the linguistic denoting relation from the unpublished manuscripts—would be indirect and mediated via <the teacher of Plato>. In the absence of this linguistic relation, the denoting phrase would retain its significance: ‘the teacher of Plato’ is significant whether or not Plato ever received instruction. However, on the view from the manuscripts we are considering, there *is* a linguistic relation—both direct and unmediated—holding between ‘the teacher of Plato’ and Socrates: the former *denotes* (in the linguistic sense) the latter. This is a significant development from the position endorsed in *PoM*.

Russell’s similarity to Frege in the short period presently under discussion continues with his treatment of *sentences* as having both meaning and denotation. The meaning of a sentence is taken to be a complex concept: the denotation of a sentence is taken to be a proposition. The meaning of ‘the table’ will be a constituent of the complex concept meant by (9); the denotation of ‘the table’ will be a constituent of the proposition denoted by (9). In adopting this position, Russell in effect endorses Frege’s ‘retrograde step’ (as Dummett (1981) considers it) of treating sentences as complex

⁷⁰ Russell shows himself well-aware of the phenomenon of ‘incomplete descriptions’: ‘*the table* (with an unexpressed addendum of the kind giving definiteness, such as “in that corner”, “at which we dined last night”, etc.) both means and denotes’ (*OMDP*: 284). This suggests that Strawson’s (1950: §III, 11) discussion of the sentence ‘the table is covered with books’ would not have worried Russell unduly. Compare *OMD*: 328-29.

singular terms. This may have seemed plausible, for a time at least, for a couple of reasons. The first is that sentences yield quite easily to nominalisation. For instance, the nominalisation of (9) yields:

(9a) The table's being black.

The second reason is that treating sentences as having both meaning and denotation enables Russell to propose a solution to the problem of 'objective falsehoods', i.e. of the subsistence of false propositions. Propositions are taken to make up the fabric of reality. If propositions are the objects of judgement, then to believe that Socrates is Greek is to stand in the *belief* relation to proposition ⟨11⟩:

⟨11⟩ ⟨Socrates is Greek⟩

But then what proposition does one believe if one believes that Socrates is Belgian? Presumably the false proposition ⟨12⟩:

⟨12⟩ ⟨Socrates is Belgian⟩

If this is the case, Socrates' being Belgian makes up part of the fabric of reality. But, surely, to say that ⟨12⟩ is false is to say that the fabric of reality *doesn't* include Socrates' being Belgian.⁷¹

It might seem then, that while 'Socrates is Greek' denotes the proposition ⟨Socrates is Greek⟩, the sentence 'Socrates is Belgian' denotes nothing. Russell's attempt to solve the problem of objective falsehoods thus begins with the recognition that he owes an account of apparently empty proper names. His suggestion is that some proper names be taken as associated with an 'improper' use (*OMDP*: 285). 'Apollo' and 'Hamlet' (and so on) are taken as substitutes for definite descriptions and, as such, to have meaning (thus explaining their significance) but no denotation.⁷² When such expressions, i.e.

⁷¹ For a fuller treatment of this problem see, e.g., Candlish 1996.

⁷² This view—a precursor to the description theory of names—surfaces in *OD* (in league with the theory of descriptions). In *OD* Russell is explicitly committed only to the view that names of mythological or fictitious characters are disguised descriptions (though he also states that we cannot have acquaintance with matter (of the kind described by physics) or with other minds (*OD*: 56), suggesting certain

those having meaning but no denotation, occur in a declarative sentence, that sentence itself has meaning but no denotation. (13) for instance, means, without denoting:

(13) The present king of France is bald.

Russell then goes on to wonder whether such an approach could be extended to sentences such as (14):

(14) Shakespeare was blind.

Here, he suggests, though there is no failure of denotation in the parts, ‘there is a failure of denotation in the whole; [. . .] the phrase should denote Shakespeare’s blindness, and [. . .] there is no such entity’ (*OMDP*: 286-287).⁷³ In this way, the problematic objective falsehoods might be jettisoned on the basis of an independently motivated philosophical account of meaning and denoting.

Now according to Levine (2005), this position is one upon which Russell, at least up until *OD*, had settled. He quotes at length from *OMDP* in support of this interpretation, urging that Russell’s view is that sentences such as (13) and (14) fail to express any proposition; he quotes, for example, the following passage:

we shall have to say that [(13)] is neither true nor false; for truth and falsehood have to do with what a sentence *denotes*, not with what it *means* [. . .]. [. . .] There is a complex concept, which is the *meaning* of [(13)]; and this concept has the form of those that denote propositions. But in the particular case considered, the concept does not denote a proposition. (*OMDP*: 286; cf. Levine 2005: 42-3)

Furthermore, he suggests, this view is continuous with the position endorsed in *PoM*. There, he suggests,

in discussing the proposition expressed by “Chimaeras are animals”, Russell concludes that we should “reject the proposition altogether” on the ground that

extensions of the view). In *OMDP* he claims that: ‘genuine proper names, when they belong to interesting people, tend to become names which have meaning’ (*OMDP*: 285). See also *OMD* (318) and *EIP* for early statements of Russell’s descriptivism about proper names.

⁷³ Cf. *DVD*: 298.

“chimaeras” expresses a denoting concept that does not denote anything. By “reject[ing] the proposition altogether”, I take Russell to be denying that it expresses a proposition at all. (Levine 2005: 43n.)

The error here is to take Russell too quickly at his unpublished word: we should be careful not to suppose Russell committed to views apparently endorsed in unpublished papers unless they are supported by strong independent evidence (textual evidence from published work, for example). In this case, it emerges in later unpublished manuscripts—*OMD* for example⁷⁴—that Russell actually abandoned the view that sentences both mean and denote.⁷⁵ I shall give his argument for the retraction presently. For the moment, I must argue against Levine’s claim that the *OMDP* view that sentences both mean and denote, and that un-denoting ones are neither true nor false, is continuous with the view of *PoM*.

Levine cites §73 of *PoM*, in which Russell considers the proposition expressed by (15):

(15) Chimaeras are animals.

Levine’s claim is that, just as in *OMDP* Russell advocates the claim that (13) is neither true nor false on the grounds that it fails to denote a proposition (this on the grounds that its subject term fails to denote), so in *PoM* he advocates the claim that (15) is neither true nor false on the grounds that it also fails to denote a proposition (on the grounds that its subject term fails to denote). However a close reading of §73 reveals that although Russell does deny that (15) expresses a proposition, he does not hold that, as it occurs in (15), ‘chimaeras’ is a denoting phrase. In §73, Russell is discussing the null-class. (The description of §73 given in the table of contents reads: ‘there are null class-concepts, but there is no null-class’ (*PoM*: xxvii).) Russell’s point concerning (15) is that although it might, from the standpoint of intension, be interpreted as asserting a relation of predicates (class-concepts), from the standpoint of extension, no satisfactory analysis is forthcoming. From the extensional standpoint, nothing is said, by (15), to be an animal. Hence:

⁷⁴ *OMDP* and *OMD* are both from 1903, but *OMD* is generally taken to be the later.

⁷⁵ Wahl (2007) also attributes to Russell the view that sentences both mean and denote, failing to recognise the retraction in *OMD*.

On the whole, it seems most correct to reject the proposition altogether, while retaining the various other propositions that would be equivalent to it if there were chimaeras. (*PoM*: §73, 74)

What is rejected in this passage is the purely extensional reading of (15), according to which ‘chimaeras’ is, in effect, a proper name for the class of chimaeras: there being no such class (taken in pure extension), there can be no such proposition as that which (15), on this reading, expresses. But ‘retaining the various other propositions that would be equivalent to [(15)] if there were chimaeras’ amounts to an admission that the following propositions are *not* to be rejected:

- ⟨15a⟩ ⟨<some chimaeras> are animals⟩
- ⟨15b⟩ ⟨<all chimaeras> are animals⟩
- ⟨15c⟩ ⟨*chimaeras* are animals⟩

That these propositions are to be retained is a simple consequence of the logical genesis view of the theory of denoting concepts I defended in Chapter One. Russell does not deny that *chimaeras* is a class-concept, and hence does not deny the associated denoting concepts derived from it. All he denies is that there is a null-class.

Thus although in the section of *PoM* under discussion Russell denies that (15) expresses a proposition, it is not for the same reason that the Russell of *OMDP* denies that (13) expresses a proposition. (15) fails to express a proposition because the (supposed) singular term ‘chimaeras’ is in fact meaningless, an empty sound. Given this, and given that Russell soon came to reject the view that (13) fails to express a proposition, we should not take the Russell of *PoM* as committed to the view that sentences both mean and denote. I turn now to Russell’s argument against this view.

Ironically, Russell’s reason for abandoning the view that sentences have both meaning and denotation was also related to the question of objective falsehoods. Consider the sentence

- (16a) *a* differs from *b*.

Nominalisation yields (16b):

(16b) The difference between *a* and *b*

On the view that sentences both mean and denote, (16a) and (16b) are taken to have the same denotation. Now (16a) certainly affirms *something*, but what it affirms cannot be what is denoted by (16b). This is because (16a) affirms something even if it is false. Yet if (16a) is false, then (16b) fails to denote (there being, in this case, no difference between *a* and *b*).

This, Russell thinks, leads to a number of difficulties (*OMD*: 326-27), the most serious of which is simply that it denies what seems to be a natural assumption, namely that:

if a proposition [sentence] denotes a fact, it seems as though the fact itself must be what we affirm [in affirming the sentence], whereas, on the view in question, we only affirm descriptions of the fact. (*OMD*: 326)

This objection embodies one of the most basic tenets of Russell's philosophy, namely that in the act of judgement one, as it were, reaches all the way out to touch reality. To judge that *Mont Blanc is more than 4000 metres high* is to enter into a direct and unmediated relation with Mont Blanc itself (snowfields and all). Yet on the view that sentences have both meaning and denotation, Russell concludes that affirmation—a form of judgement—must be a relation to a kind of complex concept, not to the proposition in question. Instead Russell retreats to a position familiar from *PoM*:

A proposition is a complex *sui generis*, and we can distinguish, in regard to it, (a) the terms *about* which it is, (b) its constituents. (*OMD*: 327)

That is to say, in judgement (or affirmation) one stands in relation to a proposition, and that proposition, if it contains any denoting concepts, may be *about* some term (or terms) not found among its constituents. But crucially, the fact that (say) the sentence (17) below may be used to affirm the happiness of Obama is due to its expressing a

proposition containing a constituent that denotes (in the logical sense) him—not due its denoting (in the linguistic sense) a proposition in which he actually occurs.

(17) The President of the USA is happy.

And supposing (17) to be true, in affirming it one stands in direct relation to a fact, namely:

⟨17⟩ ⟨<the President of the USA> is happy⟩

One does not stand in direct relation to a complex concept denoting (in the logical sense) the relevant fact.⁷⁶

If sentences are no longer taken as having both meaning and denotation, then (17)'s being about Obama is not to be explained in terms of its denoting⁷⁷ a proposition containing him. Rather (17) is about Obama in virtue of its expressing a proposition containing <the President of the USA> and this denoting concept's denoting Obama. The relation between the phrase 'the President of the USA' and Obama is very much mediated by the meaning, <the President of the USA>⁷⁸. In the primary sense, the phrase is significant solely in virtue of indicating this meaning.

We may, if we wish, still posit two linguistic relations, meaning and denoting, such that proper names denote (in the linguistic sense), and verbs and adjectives mean; but since no expressions both mean and denote (in the linguistic sense)—for recall that the relation between a denoting phrase and the entity denoted by the meaning of that phrase is *not* the direct linguistic denoting relation holding between a proper name and its bearer—the distinction could equally be captured in terms of a distinction between the kinds of terms that are standardly meant and the kinds of terms that are only denoted (in the linguistic sense). Label the former 'concepts' and the latter 'things' and we arrive back at the original position from *PoM*.

⁷⁶ A fact is just a true proposition. Note that in this discussion I assume that Russell is not committed to Hylton's (1990) 'principle of truth-value dependence'. I defend this interpretation in Chapter Four.

⁷⁷ In the linguistic sense.

⁷⁸ Wahl (1993: 79-80) agrees.

I conclude this section by emphasising the fact that, ultimately, Russell's position in the 1903-05 papers differs from *PoM* in emphasis rather than substance. He writes:

What remains to be said about denoting itself, after all these preliminaries, is little enough. The fact seems to be simply that denoting is indefinable and fundamental, that certain complexes have the property of denoting something other than themselves, and that, when such complexes are constituents of propositions, the propositions are not about the complexes, but are about what the complexes denote. This contains the whole of what I have to say on the theory of denoting [. . .]. (*OMD*: 327)

This is not different in substance from what he had said in *PoM*; and he introduces the shift of emphasis that I wish to note in the very next clause: 'it will conduce to clearness to expand and explain the two ways in which an entity may occur in a proposition' (*OMD*: 327).

4.2. *Entity- and meaning-occurrences*

In the previous section I spoke of meaning and denotation, of Shakespeare's blindness and the king of France. These, it may be thought, are some way removed from the paradoxes. This section brings the theory of meaning and denotation and the paradoxes back together.

The paradoxes involve propositions in which a class, function, predicate, propositional function (etc.), is related, in the way appropriate way for entities of that kind, to itself. What Russell comes to emphasise—especially in *OMD* and *OF*—is that the constituents of structured complexes (i.e. propositions) have different 'modes of occurrence'.⁷⁹

In Chapter One (§2.3) we distinguished 'occurrence as subject' from 'occurrence as concept'. A term occurs 'as subject' in a proposition if it may be substituted for any term at all (thing or concept) such that the resultant proposition has the same structure as the original. If the resultant proposition only has the same structure if the term is substituted for a concept, then that term occurs 'as concept'. But as we saw, the theory of denoting concepts introduces complications. For (in at least some cases) when a

⁷⁹ The relation of modes of occurrence to Russell's work on the paradoxes is also noted by Wahl (1993).

proposition includes a denoting concept, the proposition may not be about *any* of its constituents—even the one occurring in ‘subject-position’. In proposition ⟨18⟩ for example,

⟨18⟩ ⟨<the inventor of the clarinet> was German⟩

the denoting concept <the inventor of the clarinet> occurs in subject-position, and yet the proposition is not about *it* but about its denotation (Johann Christoph Denner).⁸⁰ In *OF*, which contains the most revealing discussion of modes of occurrence, Russell uses different terms for the same distinction. ‘Occurrence as subject’ becomes *occurrence as entity* and ‘occurrence as concept’ becomes *occurrence as meaning* (henceforth I adopt the new terminology).

Modes of occurrence are intimately related to the idea of *form* or *structure*. Consider propositions ⟨19⟩ and ⟨20⟩.

⟨19⟩ ⟨Bradman averaged over 50⟩

⟨20⟩ ⟨Sobers averaged over 50⟩

We immediately recognise a similarity of form in ⟨19⟩ and ⟨20⟩. This ability is, Russell says, ‘one of the pre-requisites of all reasoning from the general to the particular’ (*OF*: 366). The shared form is not a constituent of the propositions; but—at certain moments at least—Russell toys with the idea of form as an entity, a ‘mode of combination’, not unlike the ‘logical forms’ of *TK*:

A mode of combination, like everything else, is an entity; but it is not one of the entities occurring in a complex composed of entities combined in the mode in question. Thus e.g., in the case of “*A* is greater than *B*”, the mode of combination may be denoted by $\hat{x}\hat{R}\hat{y}$. This is a definite entity, but it is not a constituent of “*A* is greater than *B*”, of which the constituents are only *A*, *greater than*, and *B*. (*OnF*: 98)

⁸⁰ It may also happen that <the inventor of the clarinet> occurs in a proposition in such a way that the proposition is about it rather than Denner. Notice how the notation I use is unable to represent this distinction: ⟨<the inventor of the clarinet> is a denoting concept⟩ is—given the understanding of the notation that I have given—a false proposition about Denner. This issue—which lies at the heart of the GEA—will be taken up in Chapter Four.

Leaving the question of the ontological status of modes of combination aside, we can follow Russell's lead in isolating the shared form of (19) and (20), and represent it as follows:

$$[21] \quad {}_m[\phi]_m/{}_e[x]_e$$

(The slash '/' simply separates variables, the bracket-plus-subscript notation (e.g. ' ${}_m[$ ' and ' $]_m$ ', and ' ${}_e[$ ' and ' $]_e$ ') distinguishes the mode of occurrence of the term occurring within the brackets). In [21] whatever occurs in the position of ϕ occurs as meaning; whatever occurs in the position of x occurs as entity. As such we may speak of 'entity-position(s)' and 'meaning-position(s)' in a complex. Similarly 'entity-variation' and 'meaning-variation' are the variation of the values of the variables in entity- and meaning-position respectively.

Now the paradox of predicates, to take an example, involves the application of a predicate to itself. We have, for instance:

$$\langle 22 \rangle \quad \langle \phi(\phi) \rangle$$

$\langle 22 \rangle$, like all propositions, has a structure, a form. Indeed its form is that given in [21]. Combining notations, $\langle 22 \rangle$ amounts to $\langle 23 \rangle$:

$$\langle 23 \rangle \quad \langle {}_m[\phi]_m/{}_e[\phi]_e \rangle$$

In $\langle 23 \rangle$ we have ϕ occurring in both meaning- and entity-positions. But the difficulty, the source of which Russell now takes himself to have discovered, is this:

It seems likely that meaning-variation must be distinguished from entity-variation, and that two variables of which one *means* [i.e. occurs in meaning-position] and the other *is* [i.e. occurs in entity position] can only be equal by accident, and can't be kept equal throughout variation. (*OF*: 360, emphasis added)

That is, in (23), the two variables are intended to vary in tandem; yet because they occur in different kinds of position, their values will in general differ. Russell says:

if we assert a connection between a variable in a meaning-position and a variable in an entity-position, we must avoid denoting complexes, since these will stand for their meaning in the one position and for their denotation in the other. (*OF*: 361)

That is to say, if an entity that denotes—in particular, a variable—occurs in an entity-position, it will, in that position, yield its denotation as value; but if that same entity occurs in a meaning-position, it will not yield its denotation as value, but something else. What this other value will be is not quite clear—suffice it to note that it will be different from the value it yields in entity-position (or at least if the two values *are* the same, this is only ‘by accident’).⁸¹ Clearly, then, understanding the connection between a variable occurring as meaning and that same variable occurring as entity is going to be very important in the attempt to solve the paradoxes. For in forming a complex which genuinely applies ϕ to itself, ϕ cannot, it seems, occur both as meaning and as entity.

It may not be unreasonable to speculate that Russell hoped that the grammar of the theory of meaning and denotation would outlaw the formulation of the paradoxes in a manner analogous to that in which the grammar of the substitutional theory outlaws their formulation. This is lent support by the conjunction of the eventual failure of the theory of meaning and denotation and Russell’s later claim that his first efforts at a substitutional theory had ‘failed for want of the theory of denoting’ (Grattan-Guinness 1977: 79-80). I present this as nothing more than speculation. But whether or not Russell hoped that a more precise understanding of the theory of meaning and denotation would *outlaw* the formulation of the paradoxes, he surely recognised that it was essential for a clear statement of them. The first step towards solving a problem is

⁸¹ It seems likely that Russell supposes that the variable in meaning-position will denote its meaning. If so, variables—denoting concepts generally—*have* meanings. Yet we have been led to believe that denoting concepts *are* meanings! There is some confusion here, generated by the fact that Russell does not consider in sufficient detail the question of how a proposition can have a denoting concept as its logical subject. When, in *OF*, he does begin to seriously consider this issue, he discovers the GEA. As we shall see in Chapter Four, in the GEA Russell considers two versions of the theory of meaning and denotation: one in which denoting phrases indicate denoting complexes, which in turn *have* both meaning and denotation; one in which denoting phrases indicate denoting complexes, which are identified with meanings. These complications are not essential to any of the discussions in this or the previous chapter. They are discussed at length in Chapter Four.

often to get clear as to its proper formulation. The relation of meaning to denotation, and the role of modes of occurrence, therefore emerge as central issues in the search for a solution to the paradoxes. These issues—on which, it should be said, Russell is terribly unclear—will be probed by Russell in the GEA, and lead eventually to the demise of the theory of meaning and denotation. But this is the subject of the Chapter 4.

4.3. Intension and extension again

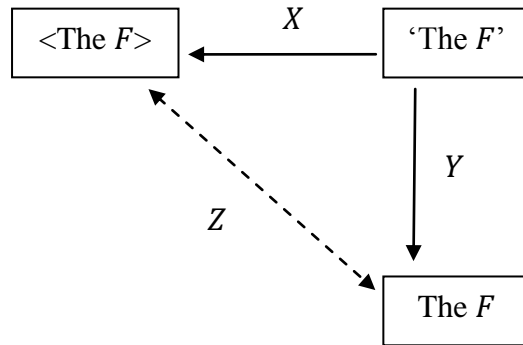
I want to conclude by briefly drawing a loose connection between the interplay of extension and intension that we encountered in relation to the ‘mixed paradox’ (§2.1), and the relation of meaning to denotation.⁸²

A class-as-many and its associated class-as-one are closely connected. Indeed, I suggested above that Russell may at times have over-estimated the extent of their connection, though he also—in more careful moments—acknowledges an ‘ultimate distinction’ between them. I attempted above to characterise their relation, and suggested that in this regard there is no avoiding a certain blurring of the lines between extension and intension. There was, I suggested, no purely extensional characterisation of the relation C (from Figure 1), holding between a class-as-many and its associated class-as-one. That relation stands, therefore, in need of investigation; for until it is clearly set out, ‘the hoary difficulties about the one and the many’ (*STCR*: 189) will remain.

We have recently set the scene for another set of equally ‘hoary difficulties’. These are the difficulties concerning the relation of meaning and denotation. Notice, to begin with, that in this field we find the same tripartite arrangement as we saw in Figure 1:

⁸² I will expand upon this in Chapter Five.

Fig. 2



The relations X and Y are linguistic: X is *meaning* and Y is *denoting* (in the linguistic sense). Relation Z is the logical relation of denoting.

The GEA focuses upon Z . It is at least arguable that similar problems as attend relation C attend relation Z . I will make some tentative proposals in this regard in Chapter Five. The point I wish to indicate here—though its force will not be felt until Chapter Four—is that the GEA challenges the theory of meaning and denotation to explain the relation between meaning and denotation. That relation cannot, ultimately, be mediated via the denoting phrase (i.e. Z cannot simply be characterised in terms of X and Y), and yet it cannot simply be taken as fundamental (both will lack sufficient explanatory value). It will be argued in Chapter Four that the GEA demonstrates the failure of the theory of meaning and denotation to account for the relation between meaning and denotation in a manner sufficiently robust to meet the requirements of a theory of denoting. The failure is attributable, I will suggest, to an inability of the theory of meaning and denotation to satisfactorily accommodate the demands imposed upon it from the dual standpoints of intension and extension. Meeting these demands involves more subtlety than the theory of meaning and denotation can accommodate. Its failure in this respect, as well as the discovery of a means of doing without it, leads Russell to abandon it. Similarly, I think, for the interplay of intension and extension in relation to classes, and those ‘hoary difficulties’ of the one and the many: once Russell found a way of doing without classes-as-many and classes-as-one, he was happy to let them go.

3. ‘On Denoting’ and the Theory of Descriptions

1. Introduction

OD marks a fundamental shift in Russell’s philosophy. Its introduction of the theory of descriptions signalled the recognition that systems of symbolism—particularly ordinary languages—are potentially highly misleading to the philosopher; it signalled a greater concern with language than had been previously evident, a concern that was to characterise all of Russell’s subsequent work. He would later write:

There is a good deal of importance to philosophy in the theory of symbolism, a good deal more than at one time I thought. I think the importance is almost entirely negative, i.e., the importance lies in the fact that unless you are fairly aware of the relation of the symbol to what it symbolizes, you will find yourself attributing to the thing properties which only belong to the symbol. (*PLA*: 185)

Language is most obviously misleading when expressions purporting to pick out an individual in fact fail to do so. In *OD* Russell was to claim that denoting phrases were of this kind: that, though every sentence in which a given denoting phrase occurs has a meaning, that phrase ‘does not, like most single words, have any significance on its own account’ (*OD*: 51). This amounted to the recognition of a class of ‘incomplete symbols’.

This chapter introduces the theory of descriptions. The theory is well-known, having found application in a good deal of subsequent analytical philosophy. However I shall emphasise the connection between the central principle of the theory—that denoting phrases are incomplete symbols—and the development of Russell’s conception of analysis between *PoM* and *OD*. The notion of an incomplete symbol, at least as Russell employs it, is intimately bound to the notion of what I shall call ‘structurally-radical interpretive analysis’ (building on certain distinctions introduced to the literature on analysis by Michael Beaney¹). The neglect of the connection between incomplete symbols and structurally-radical interpretive analysis has induced, I will argue, a tendency to mischaracterise the nature of Russell’s theory. Emphasising the connection will serve to demonstrate the distance between the theory as Russell presents

¹ For an overview of these distinctions see Beaney 2009. For an earlier discussion of different forms of analysis in Russell’s philosophy, see Weitz 1951.

and uses it, and the modified version of it advocated by many contemporary philosophers of language. The theory that Russell advocated in *OD* is not, I will argue, at all well-suited for a role in a compositional semantics of natural language. And, on the other hand, had Russell originally developed the theory advocated by contemporary philosophers of language, it is arguable that he would not have been able to use it in developing the substitutional theory of classes. This last point, however, will not be discussed until Chapter Five.

I begin (§2) with Russell's use of 'denoting'. *OD* is an extremely difficult paper. Its difficulty stems, in part, from the fact that it is not entirely clear how the word 'denoting' is to be understood. I discuss Russell's use of 'denoting', and of another key term 'about', identifying broad, general senses of these terms. §3 begins with an exploration of the role of 'decompositional' analysis in *PoM*, and its connection to the 'philosophical grammar' of that work (§3.1). I then (§3.2) introduce the notion of an 'incomplete symbol', and examine the connection between incomplete symbols, decompositional analysis, and structurally-radical interpretive analysis. The development of Russell's analytical practice is, I will suggest, more fundamental than a related development in his view, namely the introduction, in *OD*, of a syntactic (rather than semantic) criterion for being a denoting phrase. A propensity to focus undue attention upon this latter development can lead, I will argue, to failure to pay due attention to the fact that *OD* offers an account of the entire sentential contexts in which denoting phrases occur, rather than an account of those denoting phrases themselves. This distinction may seem minor, but I will show how the failure to recognise it leads to a puzzle which ought not to arise. In §4 I set out the theory of descriptions, as presented in *OD*. I will argue that because the theory involves a commitment to structurally-radical interpretive analysis, it is ill-suited to the semantics of natural language. I will argue that when modified to accommodate the demands of natural language semantics, the resultant theory is significantly different to Russell's original. I then (§5) discuss the famous logical puzzles offered in *OD*, presenting Russell's solution of them by appeal to 'scope'-ambiguity. The Chapter ends (§6) with a formulation of what I take to be the central question raised in *OD*, namely: How is it that sentences containing denoting phrases come to be about whatever it is that they are about? I will suggest that the objections that Russell presents in *OD*—to Meinong and Frege—are all objections to

rival answers to the central question. The main objection is the Gray's *Elegy* Argument (GEA). This, however, is the subject of Chapter Four.

2. Loose Use of 'Denoting' and 'About'

As in *PoM*, in *OD* Russell presses the importance of denoting. In *PoM* he had claimed:

This notion lies at the bottom (I think) of all theories of substance, of the subject-predicate logic, and of the opposition between things and ideas, discursive thought and immediate perception. (*PoM*: §56, 53)

And in the introduction to *OD* he again urges:

The subject of denoting is of very great importance, not only in logic and mathematics, but also in theory of knowledge. (*OD*: 41)

In both works, when the discussion is at its most general, Russell is best understood as using 'denoting' to mean (roughly): *that notion, whichever it is, that grounds the possibility of propositions about some collection of entities specified by way of a common property, rather than enumeration*. In this sense, the notion of denoting is inextricable bound to the notion of generality.

An advocate of the theory of denoting concepts (or the theory of meaning and denotation²) might well claim that the denoting phrase 'the father of Charles II' denotes—in some sense of that word—Charles I. For the denoting phrase indicates <the father of Charles II>, which denotes (in the technical, logical sense) Charles I. Hence sentences containing 'the father of Charles II' express propositions about Charles I. On the other hand, an advocate of the theory of descriptions might be reluctant to claim that 'the father of Charles II' denotes—in any sense of that word—Charles I. For as we shall see, on the theory of descriptions, sentences containing 'the father of Charles II' are perhaps more appropriately described as expressing propositions about a complex propositional function, rather than as expressing propositions about Charles I. However, using 'denotes' in the broad sense I wish to identify, the Russell of *OD* is quite at liberty to say:

² The theory of denoting concepts and the theory of meaning and denotation differ very little, as argued in Chapter Two.

[I]f ‘*C*’ is a denoting phrase, it may happen that there is one entity *x* (there cannot be more than one) for which the proposition ‘*x* is identical with *C*’ is true [. . .]. We may then say that the entity *x* is the denotation of the phrase ‘*C*’. Thus Scott is the denotation of ‘the author of *Waverley*’. (*OD*: 51)

Notice that the broad conception says nothing about the *mechanics* of denoting. In *PoM* the mechanics of denoting are held to involve denoting concepts and a primitive logical relation, which Russell also calls ‘denoting’.³ But this technical sense of denoting is distinct from the broader sense, and it is denoting in the broader sense to which Russell attributes, in both *PoM* and *OD*, such great importance.⁴ Since denoting in *PoM*’s technical sense is an explication of the broader notion, to attribute ‘very great importance’ to the latter *just is*, in *PoM*, to attribute it to the former. Nonetheless, the distinction between the two is clear.

It is also worth attending to the broad sense of ‘denoting’ in order to ward off a worry that might otherwise occur to one. David Kaplan attributes to the Russell of *OD* the following two views: that denoting is a notion of great importance; and that there is no logical relation of denoting, but only a linguistic one. He writes:

There is something very odd about urging the epistemological importance of *denoting* at the beginning of a work whose purpose is to show that the propositions we entertain when we *know*, *judge*, *suppose*, etc. contain *no* denoting elements. [. . .] Denoting has been [in *OD*] reduced to a property of proper definite descriptions [. . .]. It sidles into the *OD* picture through mere (and meaningless!) linguistic phrases. An ignoble end for a notion ‘of very great importance’. (Kaplan 2005: 977-78)

But there is no tension here. Denoting in the broad sense is epistemologically important (e.g. in account for the extension of knowledge beyond the realm of acquaintance), and *OD* offers an account of it; and while *OD* might be thought to signal an ‘ignoble end’

³ To recapitulate a point raised in Chapter One, the theory of denoting concepts should not be seen as an *explanation* of generality: in taking the logical relation of denoting as primitive, it assumes generality from the outset. Contrast the view of Hylton (2005d: 202-03; 2005c: 165).

⁴ When Russell says ‘the fact that description is possible—that we are able, by the employment of concepts, to designate a thing which is not a concept—is due to a logical relation between some concepts and some terms, in virtue of which such concepts inherently and logically *denote* such terms’ (*PoM*: §56, 53) I take him to be claiming that denoting (‘description’) *in the broad sense* is made possible by denoting *in the technical sense*.

for *PoM*'s logical relation of denoting⁵, this in no way constitutes an 'ignoble end' for denoting in the broad sense.

Commensurate with the broad use of 'denotes', Russell employs a broad sense of 'about'. As we shall see below, there is a sense in which, according to the theory of descriptions, the sentence 'the father of Charles II died on the scaffold' is not strictly about Charles I, but rather asserts a complex existential quantification.⁶ However accurately this represents the theory of descriptions, it does not accurately represent Russell's terminology. 'All thinking' he says, 'has to start from acquaintance; but it succeeds in thinking *about* many things with which we have no acquaintance' (*OD*: 42). That is, in thinking that the father of Charles II died on the scaffold, I think *about* Charles I, whether I am acquainted with him or not.⁷ Thus just as the Russell of *OD* is at liberty to claim that 'the father of Charles II' *denotes* Charles I, so may he claim that 'the father of Charles II died on the scaffold' is *about* Charles I.

3. Incomplete Symbols and Analysis

Russell described his philosophical method as one of *analysis*, and maintained that in approaching philosophical questions 'only by analysing is progress possible' (*MPD*: 14-15). 'Analysis' is a wide-ranging term, masking many subdivisions. For present purposes I wish to emphasise two forms of analysis employed by Russell during the period in question: 'decompositional' and 'interpretive'. The significance of Russell's discovery of a class of 'incomplete symbols' is most clearly seen in connection with the notion of a variety of interpretive analysis which I shall label 'structurally-radical interpretive analysis'. But I begin with Russell's use of decompositional analysis in *PoM*.

3.1. Decompositional analysis in *PoM*

Analysis in *PoM* is most often the *decompositional* analysis of a complex into its constituents.⁸ Typically this will involve the breaking down of a proposition to reveal its

⁵ This would be a mistake, I think. The upshot of *OD*—in particular the GEA—is that denoting complexes (i.e. complex denoting concepts) are eliminated from the ontology. But the logical relation of denoting (in *PoM*'s technical sense) arguably remains as that relation holding between the variable (now taken as fundamental) and its values. Cf. Hylton (1990: 254-56; 2007: 97).

⁶ That there is exactly one x such that x sired Charles II and x died on the scaffold.

⁷ Contrast Kaplan's 'presuppositional' interpretation of Russell (Kaplan 2005: 984).

⁸ Cf. Chapter One, §2.2.

constituents, namely the terms (entities) of which it is composed.⁹ This form of analysis is at the heart of the ‘new philosophy’¹⁰ inaugurated by Moore and Russell. Moore had written that ‘A thing becomes intelligible first when it is analysed into its constituent concepts’ (Moore 1899: 182), and that ‘we cannot define anything except by an analysis’ (in ‘the most important sense of “definition”’¹¹) (Moore 1993: 61).¹² Russell agreed, as he made clear in distinguishing mathematical from philosophical definition: ‘definition, in mathematics, does not mean, as in philosophy, an analysis of the idea to be defined into constituent ideas’ (*PoM*: §31, 27).

Russell’s use of analysis is, in a certain sense, *metaphysical*: it aims at revealing the fundamental constituents of the world. If it is to do this, the subject of analysis must be the world itself, or at least a certain part of it. Russell’s analyses are therefore analyses of propositions.¹³ But of course propositions, being unified complexes, cannot be literally broken down by analysis.¹⁴ Nor can propositions, *qua* abstract entities, be literally ‘put on’ the pages of philosophy books. Practically then, some medium is required, in which philosophical analysis can be represented. Ideally this will be a language the syntax of which has been developed with the purposes of philosophical analysis in mind. Discussing the nature of such a language, Russell later wrote:

In a logically perfect language, there will be one word and no more for every simple object, and everything that is not simple will be expressed by a combination of words. [. . .] A language of that sort will be completely analytic¹⁵, and will show at a glance the logical structure of the facts asserted or denied. The language which is set forth in *Principia Mathematica* is intended to be a language of that sort. [. . .] Actual languages are not logically perfect in this

⁹ Beany (2003) distinguishes two forms of ‘resolutive’ analysis: ‘decompositional’ analysis, typified by Russell’s analysis of a complex whole into its constituent parts (whole-part analysis); and the ‘function-argument’ analysis characteristic of Frege. Russell’s analysis of a sentence into subject and assertion might be thought of as a kind of function-argument resolutive analysis; but it should be remembered that assertions were not, for Russell, ultimate, but susceptible to further analysis—*decompositional* in nature—revealing their constituents. In this they resemble propositional functions (which, like assertions, yield to further analysis).

¹⁰ As Russell termed it (*MPD*: 54).

¹¹ This being the sense in which Moore argues that ‘good’ cannot be defined.

¹² Cf. *PoL*: §11, 18.

¹³ Cf. the claim that ‘all sound philosophy should begin with an analysis of propositions’ (*PoL*: §7, 8).

¹⁴ ‘What nature has joined together, mere philosophical analysis cannot rend asunder’ as Griffin (2007: 77) puts it.

¹⁵ As Hylton (2007: 91) points out, Russell does not mean that every sentence in the logically perfect language will be analytic (as opposed to *synthetic*), but rather that every sentence will be *fully analysed* (i.e. not susceptible to further analysis).

sense, and they cannot possibly be, if they are to serve the purposes of daily life. (*PLA*: 197-98)

A logically perfect language will be ‘transparent’ in that the grammatical properties of a sentence will correspond to logical properties of the proposition it expresses: every simple expression indicates a simple object; the way the expressions are combined in the sentence corresponds to the way in which the objects are arranged in the proposition. By the time of *PLA*—in fact ever since *OD*—Russell had come to the view that ordinary language is not at all transparent: ‘if you take [ordinary] language as a guide [to analysis] [. . .] you will be led astray’ (*PLA*: 191). However in *PoM*, as we have seen (Chapter One), he took the almost completely opposite view, stating that the grammar of ordinary language, ‘though not our master, will yet be taken as our guide’ (*PoM*: §46, 42). In this respect, the analysis of sentences of ordinary language is taken, in *PoM*, to have *metaphysical implications*.¹⁶ We may characterise the difference between the Russell of *PoM* and post-*OD* Russell in the following way. Although both hold that the analysis of a proposition is to be represented by the analysis of a sentence—of a suitable language—expressing that proposition, the Russell of *PoM* does, while the post-*OD* Russell does not, hold that ordinary language is suitable for this purpose.¹⁷

I want now to raise a point whose relevance will only become apparent later on. Certain contemporary philosophers of language—namely those who investigate the properties of a compositional semantics of natural language—are in a certain respect closer to the Russell of *PoM* than the Russell of *OD*. For the Russell of *PoM*, unlike the Russell of *OD*, demonstrates a concern that his analysis of a proposition should be more or less faithful to the form of the sentence of natural language expressing it. Of course the notions of ‘logical form’ at play in contemporary philosophy and linguistics are not equivalent to the Russellian notion. For example, Chomskyian ‘LF’ is a level of syntactical representation, unlike a Russellian proposition. But we can make the following comparison. The relationship between a natural language sentence and its LF

¹⁶ I borrow the italicised phrase from Hylton (2007: 94). I am indebted to Hylton’s excellent discussion in the present section (Hylton 2007).

¹⁷ In presenting all these points, I ignore the fact that Russell’s commitment to propositions lapses sometime around 1907; nothing of substance is materially affected. I also ignore the fact that, on the *PoM* view, one need not be committed to the claim that all logical properties of the proposition are to be *easily* discerned in the sentence: the role of tense, for example, is not obvious. The salient point is just that what *is* apparent in the grammatical form of the sentence is transparent and not misleading.

representation on the one hand, and that sentence and the proposition predicted by the philosophical grammar of *PoM* on the other, both involve a structural isomorphism. For example, both approaches hold that the logical form of a sentence of the form ‘*a* is *G*’ is, as we might put it, ‘*G(a)*’, whether ‘*a*’ is a genuine singular term *or* a denoting phrase. Suppose our sentence is of the form ‘some *F* are *G*’. According the Russell of *PoM* sentences of this form express propositions involving denoting concepts. We might represent their form thus: $G(\langle \text{some } F \rangle)$. According to, say, the system of restricted quantifiers employed by Stephan Neale¹⁸, the logical form of the sentence may be given thus: $[\text{some } x: Fx](Gx)$. Neither approach has any deep objection to the claim that the logical form of the proposition expressed by a sentence of the form ‘*a* is *G*’ (where ‘*a*’ is either a singular term or a denoting phrase) may be represented as ‘*G(a)*’, and then the respective roles of ‘*G*’ and ‘*a*’ explained. This will be of some relevance below.

3.2. *Incomplete symbols and ‘structurally-radical’ interpretative analysis*

Analysis in *PoM* is primarily a matter of breaking a sentence down to its constituents, and then pairing each constituent of the sentence to a constituent of the associated proposition. Let us call the propositional constituent to which a subsentential expression is ‘paired’ its *propositional complement*.¹⁹ Russell’s dictum that ‘*Words* all have meaning, in the simple sense that they are symbols which stand for something other than themselves’ (*PoM*: §51, 47) then amounts to the claim that every significant subsentential expression has a propositional complement. With this terminology onboard we may say that what the Russell of *OD* comes to recognise is that an expression can occur significantly in a sentence *without* having a propositional complement. As he would later put it, denoting phrases are ‘incomplete symbols’.

- *Fundamental Principle of OD*: Denoting phrases are incomplete symbols.

But what exactly is an incomplete symbol?

In *PM* Whitehead and Russell write:

¹⁸ See, e.g., Neale 1990; 1993.

¹⁹ E.g. Socrates is the propositional complement of ‘Socrates’; *redness* is the propositional complement of ‘red’; and $\langle \text{the teacher of Plato} \rangle$ is the propositional complement of ‘the teacher of Plato’ (on the theory of denoting concepts).

By an ‘incomplete’ symbol we mean a symbol which is not supposed to have any meaning in isolation, but is only defined in certain contexts. (*PM*: 66)

And they contrast incomplete symbols with proper names:

‘Socrates’, for example, stands for a certain man, and therefore has a meaning by itself, without the need of any context.²⁰ (*PM*: 66)

We may, if we like, label proper names ‘complete’ symbols. But to do so is perhaps to encourage a misunderstanding. Frege drew a sharp distinction between *objects* and *functions*. Functions, unlike objects, harbour a ‘gap’, an argument place into which an object can be fitted, to yield a value of that function for that argument. At the symbolic level the incompleteness of functions is represented either by leaving a gap in the symbol for a function, or by using a Greek letter, e.g. ‘() + 7 = 12’, or ‘ ξ is greater than 27’. Such expressions may then be said to be ‘incomplete’, their incompleteness mirroring an incompleteness in the world. Correspondingly, the completeness of objects is mirrored in the completeness of proper names. Now although Russell’s proper names may be complete in the same sense as Frege’s, his incomplete symbols are not incomplete in Frege’s sense. In the Fregean sense, a symbol is incomplete if it stands for an incomplete kind of entity. But for Russell, an incomplete symbol stands for nothing at all.²¹

This point is worth emphasising since it seems to discredit a certain interpretation of Russell’s position. Alexander Miller (1998: 62), for example, attributes to Russell the view that definite descriptions have *second-level functions* for their semantic values, and so accuses him of having mis-described his own position (Miller 1998: 311n.).²² But Russell’s view *is* that descriptions have no significance on their own account. This is what it means to say that ‘denoting phrases never have any meaning in themselves’ (*OD*: 43). To suppose that *Russell’s* theory involves the claim that denoting phrases

²⁰ Russell does not really think that ‘Socrates’ is a genuine proper name (at least certainly not on the lips of anyone other than Socrates himself).

²¹ A further difference is that, for Frege, incompleteness is called upon in explanation of the unity of the proposition (cf. Frege 1892b: 193). Quite how much of a difference this ultimately amounts to is questionable (cf. Gaskin 2008).

²² Gandon (2007) demonstrates how one might follow through this proposal, though he does not attribute the claim to Russell.

have second-level functions for their semantic values, is, it seems to me, analogous to supposing that his theory involves, say, a Gricean theory of utterance interpretation. Such modifications may be implemented—and rightly so, for certain purposes—but they *are* modifications, and ought not to be read back into Russell’s position.

Russell’s position was that denoting phrases are incomplete symbols, having no propositional complements. Such symbols occur frequently in natural languages²³ and it is fortunate that they do, for otherwise sentences might be enormously long and unwieldy. Such exigencies also guide the construction of a logically perfect language. Hence the language of *PM* includes incomplete symbols in order to shorten its formulae. Such expressions as ‘ $(\iota x)(\phi x)$ ’ are given a ‘contextual definition’ however:

we must not attempt to define “ $(\iota x)(\phi x)$,” but must define the *uses* of this symbol, *i.e.* the propositions in whose symbolic expression it occurs. (*PM*: 67)²⁴

It follows from the fact that incomplete symbols have no propositional complements, that if an incomplete symbol is to be defined at all, it must be given a contextual definition. That definition will specify the manner in which the incomplete symbol may be eliminated from the symbolism in which it occurs.

The details of Russell’s theory of denoting phrases as incomplete symbols must wait until the following section. But even now we are in a position to begin to appreciate the radical alteration that *OD* brings to his view. In *PoM*, although ordinary language was not held to be logically perfect (grammar being only the guide, not the master), it was nonetheless deemed sufficiently transparent for the purposes of philosophy. Ordinary language and use might suggest that, say, ‘the teacher of Plato’ stands for Socrates; and while the theory of denoting concepts reveals that language is misleading in *this* regard (since ‘the teacher of Plato’ stands for <the teacher of Plato>, not Socrates), it is at least trustworthy in providing *something* for the phrase to indicate. By *OD* however, Russell’s view is that ordinary language is thoroughly unsuitable for philosophy: denoting phrases, though having the character of substantives, in fact fail to indicate any entity whatsoever.

²³ Particularly if one is, like Russell, a descriptivist about (merely) grammatically proper names.

²⁴ Hence: *14.01: $[(\iota x)(\phi x)] \cdot \psi(\iota x)(\phi x) \cdot = : (\exists b) : \phi x \cdot \equiv_x \cdot x = b : \psi b$ Df (*PM*: 173).

Decompositional analysis in *PoM* proceeded on the assumption that every significant subsentential expression had a propositional complement. That assumption lapsing in *OD*, the decompositional analysis of a sentence must now be *preceded* by a preliminary analytic process which purges the sentence of any incomplete symbols. This process may itself be many-staged. If the sentence contains any proper names, these may have to be recast as the definite descriptions that they disguise (assuming one endorses such descriptivism), before being eliminated as incomplete symbols.²⁵ But however many of these stages there be, they must all be carried through *prior to* the decompositional stage of the analysis.²⁶ Immediately prior to the decompositional stage of the analysis, one should arrive at a sentence in something approximating a logically perfect language (at least in the sense that every subsentential expression will have a propositional complement). What I should like to stress is that the need for this initial stage of analysis—this essential preliminary to the decompositional stage—is intimately bound up with the notion of an incomplete symbol.²⁷

This initial stage will primarily involve what Michael Beaney calls ‘interpretive analysis’.²⁸ In the broadest terms, interpretive analysis ‘involves “translating” something into a particular framework’ (Beaney 2007c: 198). Beaney cites the Cartesian development of analytical geometry, which allowed for solutions to geometrical problems by ‘translating’ them into algebraic terms, as a prime example.²⁹

Beaney (2003: 156ff) distinguishes ‘paraphrastic’ and ‘reductive’ forms of interpretive analysis. Paraphrastic interpretive analysis involves the rephrasal of the

²⁵ Similarly, scope ambiguities may have to be resolved. I discuss scope in §5 below.

²⁶ This process may involve reference to the syntactic form of expressions (e.g. for identifying denoting phrases), or perhaps a recognition that some descriptions of the appropriate syntactic form are not apt for the theory to handle (cf. Moore’s examples ‘the heart pumps blood into the arteries’, ‘the lion is the king of beasts’ and so on). Recognising that a proper name is a disguised incomplete symbol might involve epistemological considerations (such as whether one could possibly be acquainted with the purported bearer of the purported name). In such respects as these, the process may be ‘epistemologically driven’, as Hylton (2005c: 167) puts it. Hylton has repeatedly stressed the role of epistemological factors in constraining and guiding the analytical process (e.g. Hylton 1996; 2007). He is quite right to do so. Epistemological issues are not the main focus of this thesis however, so I have not given them the attention that, in a larger study, they would certainly warrant.

²⁷ Cf. Hylton (2005d: 204): ‘The idea of an incomplete symbol made an immense difference to Russell’s thought. [...] After [OD], Russell’s idea of analysis is quite different’.

²⁸ Beaney 2003; 2007a; 2007b; 2007c; 2009. Sometimes it is referred to as ‘transformative’ or ‘explicatory’.

²⁹ This is not to say that it may not also be an example of other forms of analysis. Distinctions among forms of analysis are not exclusive.

analysandum such that its logical form is more clearly displayed, but carries with it no ‘positive’ metaphysical commitments. Reductive interpretive analysis will involve rephrasal, but will go ‘a step further in aiming to reveal “deep structure” and “ultimate constituents”’ (Beaney 2003: 156). In his article, Beaney cites Frege’s analysis of existential statements in terms of the second-level predicate ‘ ξ is instantiated’ as an example of (mere) paraphrastic analysis, and the theory of descriptions as an example of reductive analysis. Beaney’s distinction suits his purpose of comparing Frege and Russell, but it will not suffice here: the *PoM* analyses of sentences containing denoting phrases may be cast as *both* paraphrastic and reductive. This quashes any hope of characterising the development of Russell’s style of analysis between *PoM* and *OD* in such terms. To illustrate with an example, a defender of the theory of denoting concepts might analyse sentences of the form ‘some *F* are *G*’ as expressing propositions of the form ‘ $G(<\text{some } F>)$ ’. This would count as paraphrastic, for there is certainly a rephrasal; and also counts as reductive, since it aims to reveal the ultimate constituents of the proposition. But what *we* require is a form of analysis that can be seen as applying to *OD* but not to *PoM*. Neither paraphrastic nor reductive interpretive analysis will fit this bill.

As an alternative I propose that we take the form of analysis associated with Russell’s account of incomplete symbols as essentially involving a *restructuring* of the sentence to be analysed. Here ‘restructuring’ is to be understood as stronger than mere rephrasal. We intuitively recognise the difference between sentences of (say) subject-predicate form (i.e. ‘*Fa*’), relational form (i.e. ‘*aRb*’), and existentially quantified form (e.g. ‘ $(\exists x)(Fx)$ ’): I will describe an analysis as involving restructuring if the analysandum has one of these forms, and the analysans another. This kind of analysis is paraphrastic in Beaney’s sense, but unlike merely paraphrastic analysis, it is here *guaranteed* that analysandum and analysans are of different form. For the sake of having a name, let’s call this form of analysis ‘structurally-radical interpretive analysis’, or ‘SR-interpretive analysis’ for short.

By treating denoting phrases as incomplete symbols, Russell commits himself to SR-interpretive analysis for sentences involving denoting phrases. In ordinary language, denoting phrases are substantives. Hence the sentences in which they occur are of either subject-predicate or relational form. But, given the details of the theory of descriptions

(to be set out presently), all sentences involving denoting phrases will now be taken to express propositions having quantificational forms.³⁰ Thus it emerges as a significant difference between the *OD* and *PoM* theories of denoting, that while the former *does*, the latter does *not*, involve a commitment to SR-interpretive analysis.

I am claiming that by endorsing the fundamental principle of *OD*—that denoting phrases are incomplete symbols—Russell commits himself to the claim that the decompositional analysis of a sentence containing a denoting phrase must be preceded by a process of SR-interpretive analysis designed to eliminate any incomplete symbols, thereby recasting the sentence in a radically different structure. The subject of SR-interpretive analysis will always be an entire sentence (rather than an isolated phrase). But in order to discern those sentences that require SR-interpretive analysis, one must have some method of recognising denoting phrases within sentences. That is, there must be some criterion by which an expression counts as a denoting phrase, and by which one may recognise it as such.³¹ In this regard it is often noted that in *OD* Russell offers a syntactic criterion for denoting phrases: ‘A phrase is denoting solely in virtue of its *form*’ (*OD*: 41). This is to allow that the significance of a denoting phrase is independent of its descriptive condition being satisfied or of its indicating anything (a denoting concept, for instance). This is sometimes presented as a development from the position in *PoM*. Thus Kaplan:

In *PoM*, a phrase was said to be a ‘denoting phrase’ because it was a phrase that denoted.³² Now, a phrase will be said to be a ‘denoting phrase’ whenever it has the appropriate syntactical form. No semantic matters are presupposed. (Kaplan 2005: 969)

³⁰ To be strict, ‘form’ here is *syntactic* form, rather than *logical* form, since one might claim that ⟨the *F*⟩ is of quantificational form in virtue of containing a denoting concept. In the sense of ‘form’ currently under discussion, I intend that such a proposition be understood as having subject-predicate form.

³¹ This arguably means that SR-interpretive analysis—or almost any interpretive analysis at that—will never be *purely* interpretive, but will also involve some other form of analysis. In the present case, a kind of initial decompositional analysis of the sentence is applied in order to discern the significant subsentential expressions. One may distinguish forms of analysis without claiming that they are wholly independent of one another. Hence Beaney: ‘in actual practices of analysis, [various] modes are typically combined’ (Beaney 2007c: 197).

³² Russell did not hold that denoting *phrases* denote *in the technical sense* (only denoting concepts do that). Kaplan is here aping Russell’s familiar carelessness in separating linguistic items from the entities they indicate. More carefully, a phrase is a ‘denoting phrase’ if it indicates a denoting concept.

For those of us concerned to understand Russell's developing views on denoting, this appears to be an important point.

A number of points should be raised in response, however. Firstly, I am not aware of any passage in *PoM* in which Russell gives a criterion for denoting phrases (and Kaplan does not cite one). Russell gives a criterion for being a denoting *concept*, but that is another matter. Secondly, the observation that every class-concept is guaranteed an associated array of denoting concepts derived from it³³, suggests that the position in *PoM* is not incompatible with a syntactic criterion. One might say that a phrase is a denoting phrase if it consists of one of the determiners ('all', 'every', 'any', etc.) annexed to an expression for a class-concept. What denoting concepts there are is determined by what class-concepts there are: hence if '*F*' is a name of a class-concept, 'det *F*' ('det' for 'determiner') is certain to indicate a denoting concept. A semantic matter is presupposed here—namely that '*F*' indicates a class-concept—but this is equally a presupposition of the *OD* position. These two points suggest that Russell *might* have intended a syntactic criterion in *PoM*. In fact I doubt that this is the case, for the simple reason that Russell was not so concerned with language in *PoM* that the matter was likely to have occurred to him. If pushed, he would most likely have offered the kind of semantic criterion that Kaplan supposes. But the interesting point is that even if this is so, the development from the semantic to the syntactic criterion is subordinate to a more fundamental development, namely the recognition of a class of incomplete symbols. If denoting phrases 'are not assumed to have any meaning in isolation' (*OD*: 42) then it follows as a matter of course that they are not to be identified by appeal to the entities they indicate. Thus had we chosen to interpret the Russell of *PoM* as endorsing a semantic criterion, that criterion could not have applied to the *OD* theory. But such a development would be premised upon, and so secondary to, the more fundamental development, namely Russell's coming to endorse the fundamental principle of *OD*.

Failure to recognise that the development of a syntactic criterion is secondary to the discovery of the fundamental principle of *OD* encourages one to focus unduly on denoting phrases, rather than on the sentences in which they occur. What *OD* provides

³³ Cf. the 'logical genesis view' defended in Chapter One.

is a theory of the analysis of the sentences in which denoting phrases occur, not, strictly speaking, of the denoting phrases themselves. Unfortunately, Russell was not very clear in marking out this distinction. For instance, in the opening paragraph of *OD* he tells us what he means by ‘denoting phrase’ and then indicates that the problem to be addressed is ‘the interpretation of such phrases’ (*OD*: 41). Or again, turning from indefinite to definite descriptions, he writes ‘It remains to interpret phrases containing *the*’ (*OD*: 44). However, in (what will appear to anyone of my view as) a more careful mood, he summarises his theory as giving ‘a reduction of all *propositions* [i.e. *sentences*] in which denoting phrases occur to forms in which no such phrases occur’ (*OD*: 45 emphasis added). It is also plain that the definitions of denoting phrases he provides are all contextual definitions (as discussed in the following section).

The opposition between giving an account of denoting phrases and giving an account of the sentences in which such phrases occur may seem minimal.³⁴ Perhaps it seems all the more minimal for the influence of contemporary philosophy of language. For in constructing a compositional semantics of natural language it is imperative to speak to the meaning of all significant subsentential expressions, denoting phrases included. But asking after the meaning, or the interpretation, or the proper analysis of a denoting phrase—rather than the sentences in which it occurs—is what *OD* aims to warn us against (whether Russell was clear on this matter or not). The point is important, and will recur later on. It can be illustrated by considering a certain puzzle discussed by Stephen Neale.

Neale describes Russell as aiming to provide a ‘theory of how descriptions bear on propositional content’ (2005: 817). As an interpretation of Russell this is, I have claimed, misleading and to be avoided. But it is natural that Neale should put things this way, since he has produced a celebrated defence of a Russellian³⁵ theory of descriptions

³⁴ One discerns in the literature a failure to adhere strictly to the distinction. For example Hylton describes the theory of descriptions as ‘a method of analyzing definite descriptions’ (2005d: 185). This is not the best way of putting things. Note that later on he describes the theory as ‘a method of analyzing complete sentences’ (2005d: 187)—a much better way of putting things. My point is not that the first quotation gets Russell *wrong* exactly, but that it is potentially misleading and that the second quote is by far to be preferred. See also Gandon (2007: 117): ‘Incomplete symbols have a logical content which is displayed by the contextual definition’. This is not *wrong* exactly, but it is misleading: it isn’t the incomplete symbol that has logical content, but the entire context.

³⁵ Only *Russellian* since, as I will argue in §4, the theory Neale defends differs significantly from Russell’s official view.

as a contribution to the semantics of natural language.³⁶ The puzzle is outlined in a footnote. I quote at length.

Before we even get to the details of Russell's positive proposal, there is a serious issue about how to *describe* what it is that Russell is going to give us. Suppose Russell provides a perfectly good description that uniquely specifies the contribution 'the ϕ ' makes to the proposition expressed by ' $C(\text{the } \phi)$ ', the following for example:

- (i) the contribution 'the ϕ ' makes to the proposition expressed by ' $C(\text{the } \phi)$ '.

Suppose we name whatever it is that (i) describes ' m '. If a name's meaning is just its bearer, then whatever it is that (i) describes, namely, m , is the meaning of ' m ', and thereby the contribution ' m ' makes to the proposition expressed by ' $C(m)$ '. So now ' $C(m)$ ' and ' $C(\text{the } \phi)$ ' seem to express the same proposition, for surely C 's contribution is constant! Russell refuted before we even get to the details of his proposal! (Neale 2005: 817n.)

One's immediate reaction here is to point out that there can be no genuine name ' m '; for if descriptions are incomplete symbols, (i) fails to describe anything (hence there is nothing for ' m ' to name). This is a point Neale immediately makes, and so the 'puzzle' ought to be dismissed here. Nevertheless he continues:

But why should our use of descriptions (and our own powers of description) be so curtailed, **given that Russell aims to show us *the way 'the ϕ ' bears on propositional content?*** – don't the italicized words before the dash constitute a definite description? Ditto '*how "the ϕ " bears on propositional content*', '*the relation between "the ϕ " and the identity of the proposition expressed by " $C(\text{the } \phi)$ "*', and even '*how "the ϕ " works*'. There seems to me to be a genuine puzzle here, not unrelated to the one Russell is discussing in the Gray's Elegy passages. (Neale 2005: 817-18n., bold emphasis added)

Now it is true that the various descriptions Neale offers are all, on Russell's view, empty. And if Russell's aim in *OD* really were to give an account of the way 'the ϕ ' bears on propositional content, then the fact that the description 'the way "the ϕ " bears

³⁶ Elsewhere in his paper Neale's expresses himself in a way compatible with my interpretation of Russell. My claim is not that Neale is consistently wrong on this matter, but rather that the issue is not clearly addressed—indeed most often missed—and that, as such, there is a tendency to equivocate. Neale is not alone here: the same phenomenon occurs elsewhere in the literature (cf. n. 34).

on propositional content' is empty would be puzzling (though I doubt the puzzle has anything to do with the Gray's *Elegy* passages). But as I have stressed, Russell's aim is *not* to provide a 'theory of how descriptions bear on propositional content'. What he provides is a method for applying an SR-interpretive analysis to any sentence containing a denoting phrase. To ask after the contribution of some particular phrase to the proposition expressed by the sentence in which it occurs, is to be more in tune with the Russell of *PoM* than *OD*. It is to engage with a 'picture of analysis [. . .] which will go word by word, or phrase by phrase, rather than sentence by sentence'—this quotation is from Hylton (2005c: 164), describing analysis in *PoM*. I suggested above (§3.1) that some contemporary philosophers of language are in certain respects closer to the Russell of *PoM* than the Russell of *OD*. This, I would suggest, is one of those respects.³⁷

4. The Theory of Descriptions

The conclusions of the previous section were these. Analysis in *PoM* had been primarily a matter of breaking down (decomposing) a sentence to its constituents, and then pairing each of them up with a constituent of the proposition expressed by that sentence. *OD* brings with it significant changes. In endorsing the fundamental principle of *OD*—that denoting phrases are incomplete symbols having no meaning in isolation from the sentential contexts in which they occur—Russell commits himself to the view that the kind of compositional analysis at work in *PoM* must be preceded by a stage of SR-interpretive analysis. This stage of analysis will take a sentence containing a denoting phrase and reformulate it, thereby attributing to it a radically different structure, and eliminating the incomplete symbols occurring therein. Russell should therefore be seen as offering, in *OD*, a theory of the analysis of the sentences in which denoting phrases occur, rather than of the denoting phrases themselves, considered in isolation. Let us turn now to the details of that theory.

³⁷ Throughout the puzzle Neale, in effect, gives vent to a kind of bewilderment: after all, the description (*i*) seems a perfectly standard description for a philosopher to ponder. To an extent I share this sense of bewilderment. However, what is really bewildering (I suspect) is not so much that Russell should endorse the view that I attribute to him (on which the puzzle is not genuine), but that his work should be at such a remove from contemporary concerns in the philosophy of language.

Russell takes as ‘ultimate’ and ‘fundamental’ the notions of the variable and of a propositional function $C(x)$ being true for all values of x (*OD*: 42).³⁸ He then offers the following contextual definitions of the most primitive denoting phrases ‘everything’, ‘nothing’ and ‘something’ (*OD*: 42):

- | | | | |
|-------|------------------------|-----|--|
| (i) | $C(\text{everything})$ | ... | ‘ $C(x)$ is always true’; |
| (ii) | $C(\text{nothing})$ | ... | “‘ $C(x)$ is false’ is always true”; |
| (iii) | $C(\text{something})$ | ... | ‘It is false that “‘ $C(x)$ is false’ is always true.” |

Moving to less primitive denoting phrases, and taking the predicate *human* to define the class of men, Russell offers the following contextual definitions (*OD*: 43-4):

- | | | | |
|--------|---------------------------|-----|--|
| (iv) | ‘ $C(\text{all men})$ ’ | ... | ‘If x is human, then $C(x)$ is true’ is always true; |
| (v) | ‘ $C(\text{no men})$ ’ | ... | ‘If x is human, then $C(x)$ is false’ is always true; |
| (vi) | ‘ $C(\text{some men})$ ’ | ... | same as ‘ $C(\text{a man})$ ’; |
| (vii) | ‘ $C(\text{a man})$ ’ | ... | It is false that ‘ $C(x)$ and x is human’ is always false; |
| (viii) | ‘ $C(\text{every man})$ ’ | ... | same as ‘ $C(\text{all men})$ ’. |

(Russell’s switching, in (vii), from ‘is always true’ to ‘is always false’ is sloppy but ultimately unproblematic.)

Sentences containing definite descriptions are the subject of the bulk of *OD*. Their interpretation is not, in essentials, very different from the above.

- | | | | |
|------|-------------------------|-----|--|
| (ix) | ‘ $C(\text{the man})$ ’ | ... | It is false that ‘ x is human and $C(x)$, and it is always true of y that if y is human, y is identical to x ’ is always false. ³⁹ |
|------|-------------------------|-----|--|

More informally, Russell would later express this by saying that the sentence ‘the author of *Waverley* was Scotch’ ‘involves’ the following claims (*IMP*: 177):

³⁸ In taking the variable as fundamental Russell assumes the notion of generality. This notion had also been assumed in *PoM*, where the logical relation of denoting is taken as fundamental. Cf. n. 3 above; also Chapter One, §4.2.2.

³⁹ Recovered from *OD*: 43-4.

- (a) “*x* wrote *Waverley*” is not always false;
- (b) “if *x* and *y* wrote *Waverley*, *x* and *y* are identical” is always true;
- (c) “if *x* wrote *Waverley*, *x* was Scotch” is always true.

This is perhaps clearer, but it should be remembered that although the proposition expressed ‘involves’ (in some sense of this word) these individual claims, it *contains* a single propositional function (rather than three distinct propositional functions conjoined). Commentators are not generally particularly clear on this point. Hylton, for example, attributes to Russell the claim that ‘the President of the USA in 1999 is a Democrat’ expresses a proposition which says, of the property *being President of the USA in 1999*,

that one and only one thing satisfies it or falls under it (and that thing is a democrat). That is how it gets to be (indirectly) about Bill Clinton: by being (directly) about a property which he and only he satisfies. (Hylton 2005d: 202-203)^{40 41}

But compare the following two Russellian claims:

There are, in the last analysis, only two things that can be done with a propositional function: one is to assert that it is true in *all* cases, the other to assert that it is true in at least one case, or in *some* cases. (*IMP*: 158)

When we say ‘there are men,’ that means that the propositional function ‘*x* is a man’ is sometimes true. When we say ‘some men are Greeks,’ that means that the propositional function ‘*x* is a man and a Greek’ is sometimes true. (*IMP*: 159)

⁴⁰ See also Jacquette & Griffin 2009: 7; and Lackey in Russell 1973: 19.

⁴¹ I hope I am not unfair to Hylton here. The passage seems to me to imply that Hylton attributes to Russell the following view: that ‘the President of the USA in 1999 is a Democrat’ says of the property *being President of the USA in 1999* that one thing falls under it, and that, whatever that thing is, it also falls under the property *being a democrat*. Whereas Russell’s view as I understand it is that ‘the President of the USA in 1999 is a Democrat’ says of the property *uniquely being President of the USA in 1999 and being a Democrat* that at least one entity falls under it.

It follows that when we say ‘the President of the USA in 1999 is a Democrat’, we assert a proposition to the effect that the propositional function ‘ x and nothing else is a President of the USA in 1999, and x is a Democrat’ is true in at least one case.⁴²

This is important because it forces the conclusion that in a sentence of the form (1a), the predicate expression ‘is G ’ functions in a manner analogous to an incomplete symbol.

(1a) Some F is G .

The explanation is as follows. According to the theory of descriptions (1a) expresses a proposition whose form is more nearly captured by (1b) (locutions (i)-(ix) are cumbersome so I employ more familiar notation):

(1b) $(\exists x)(Fx \ \& \ Gx)$

The propositional function occurring in (1b)— $F\hat{x} \ \& \ G\hat{x}$ —is too complex to be a plausible candidate for the propositional complement of the predicate ‘is G ’. But what else could plausibly be the predicate’s propositional complement? The predicate has no propositional complement here, though it will have one when its union with a genuine proper name forms a sentence. In reflection of this ambiguity, we might term predicate expressions ‘quasi-incomplete symbols’.

Although this is an unfamiliar way of looking at things, it is actually to be expected. Compare the following sentences (and take ‘Socrates’ to be a genuine proper name):

(2) Socrates is Greek.

(3) The teacher of Plato is Greek.

The removal of ‘Socrates’ from (2), corresponds to the removal of one of the constituents of the proposition (2) expresses—namely Socrates; and removing Socrates from $\langle 2 \rangle$ (i.e. the proposition expressed by (2)) leaves behind (so to speak) the

⁴² I ignore that fact that, as I have phrased it, the propositional function here contains denoting phrases (‘ a President of the USA in 1999’, ‘ a Democrat’): this is an accident of English.

propositional constituent corresponding to the predicate expression. But because denoting phrases are incomplete symbols, the removal of ‘The teacher of Plato’ from (3) does not correspond to the removal of one of the constituents of $\langle 3 \rangle$. Hence the predicate ‘is Greek’, as it occurs in (3), cannot be made to correspond to what is left over when the propositional complement of the description is removed from $\langle 3 \rangle$ —for there is no such thing. (A similar kind of procedure is implemented by G. E. Moore. He argues for the claim that descriptions are incomplete symbols (at least sometimes) by noting that if we remove ‘is wise’ from ‘the king of France is wise’, and then from ‘there is somebody or other of whom it is true that he is a king of France, that nobody else is so, and that he is wise’, we do not end up with two expressions having the same meaning (Moore 1951: 219ff).)

A simple informal argument will serve to bring out the point. Suppose we have a sentence ‘ AB ’, composed of two subsentential expressions ‘ A ’ and ‘ B ’, where ‘ A ’ is an incomplete symbol. And suppose that ‘ AB ’ expresses a proposition $\langle XYZ \rangle$ composed of three constituents X , Y , and Z . Since ‘ A ’ is an incomplete symbol, none of X , Y , or Z , nor any combination of them, is its propositional complement. Let us now ask: Could ‘ B ’ have a propositional complement? We will find that it could not. For suppose it did have a propositional complement, the compound YZ say. Nothing would stop us assigning X , the remaining constituent of the proposition, as the propositional complement of ‘ A ’. But that would contradict the assumption that ‘ A ’ is an incomplete symbol. Hence it follows that *either* the propositional complement of ‘ B ’ is the entire proposition $\langle XYZ \rangle$, so that there is nothing ‘left over’ for ‘ A ’ to correspond to, *or* ‘ B ’ has *no* propositional complement and is a quasi-incomplete symbol. The first option is intolerable, hence we conclude that ‘ B ’ is a quasi-incomplete symbol.

On the interpretation that I am proposing, Russell has a slightly strange view of the relation between the surface grammatical form of a sentence and logical form of the proposition it expresses. On my interpretation, Russell holds that in a sentence of the form ‘the F is G ’ the definite description is an incomplete symbol and the predicate expression a quasi-incomplete symbol; hence the relation between the syntactic properties of the sentence and the logical properties of the proposition become a little obscure: logical properties cannot, for example, be simply ‘read off’ from the sentence’s

grammatical properties, and we cannot legitimately ask after the contribution of either the subject or predicate expressions, without inducing Neale's puzzle from above. These consequences are unexpected, but I would cite a mitigating factor. The reason they are unexpected is arguably at least partly due to the tendency of the analytical tradition to pay insufficient attention to its own history. Russell's great insights in this area have been influential in virtue of having been put to work in subsequent philosophy. Their origins and original purpose have not been the main focus of attention; almost inevitably, contemporary analytical philosophers see the theory of descriptions through contemporary analytical philosophers' eyes. Hence any interpretation according to which Russell's familiar old theory of descriptions is cast as unfamiliar and somewhat remote from contemporary concerns is liable to strike us unexpected.

If it follows from Russell's official view that predicate expressions are quasi-incomplete symbols, and so ambiguous⁴³, this would seem to provide a compelling reason for contemporary philosophers of language to modify Russell's theory. In a compositional semantics of natural language it would be better if predicate expressions were not ambiguous in this way. The modification is easily effected through the introduction of restricted quantifiers (see e.g. Neale 1990; 1993)—though it is never introduced on the grounds that I am recommending. As discussed below, Neale claims that his employment of restricted quantifiers is a matter of choice (1990: 45). But as I see it, the modification is in fact *forced*. This being so, contemporary philosophers of language endorse a theory which differs from Russell's official version in a noteworthy respect. This, we shall see in Chapter Five, is of some consequence. For the theory as modified by contemporary philosophers of language is in fact unable to provide the support that the 'substitutional theory' requires. Thus Russell's attempts to use the theory of descriptions to disarm the paradoxes relies upon his employment of the original, official version of the theory, rather than the modified version.

Neale claims that to cast the theory in terms of restricted quantifiers extends its range of application by allowing for the treatment of determiners, such as 'most', which otherwise resist interpretation. He says that to do so:

⁴³ Ambiguous because they interact differently with denoting phrases and (genuine) proper names.

would *not* be to present an *alternative* to the Theory of Descriptions; it would be to choose a language other than that of *Principia Mathematica* in which to state and apply the theory. (Neale 1990: 45)

But it is not really the language in which we express the theory that is at issue. What is at issue is the logical form of the proposition expressed by a sentence of the form

(4) The F is G .

For Russell the logical form involves a certain propositional function and the assertion that it is sometimes true. As we might put it:

(4_R) $(\exists x)(Fx \ \& \ (y)(Fy \rightarrow y = x) \ \& \ Gx)$

Whereas, for Neale, the logical form is this:

(4_N) [the $x: Fx$](Gx)

These are equivalent (Neale 1990: 45). But—to make the point informally—(4_N) appears to say that something is G (namely whatever is uniquely F), while (4_R) says that something is *uniquely- F and G* . That is to say, it appears to be the case that Neale sanctions the representation of the logical form of (4) as, in effect, ‘ $G(\text{the } F)$ ’—at least as a preliminary to the more accurate (4_N). This reflects that fact, mentioned a couple of times above, that in some respects, contemporary philosophy of language is closer to the Russell of *PoM* than the Russell of *OD*. Or to put the same point another way: (4_R) reflects, but (4_N) does not reflect, the fact that the predicate ‘is G ’ is a quasi-incomplete symbol. This poses a kind of dilemma. On the one hand, it may be that (4_N) is requiring further analysis, which will yield something of the form of (4_R). (I take it that this is Neale’s position, though he would not necessarily use the notation of (4_R).) But if so, the problem raised by the ambiguity of predicate expressions remains. Yet if, on the other hand, (4_N) does not require further analysis, then though the problem of the ambiguity of predicate expressions is resolved, Neale’s theory differs significantly from Russell’s. The point, let me stress, is not that there is anything *wrong* with Neale’s

theory⁴⁴, but simply that—to the extent that it relieves the worry about the ambiguity of predicates—it is not Russell’s.

I raise this point concerning Neale’s position for two reasons. Firstly, it is, I think, independently quite interesting. Secondly, in drawing out the reason why his position is not obviously Russell’s, we sharpen our understanding of Russell’s position. Let us continue by considering a response one might pursue. One might claim that even if I am correct that (4_R) and (4_N) express (by Russell’s standards) different propositions, Neale nonetheless preserves enough of what is distinctive about Russell’s view for his theory to count as a modification of, rather than an alternative to, Russell’s theory. After all, descriptions are still, on Neale’s view, incomplete symbols, or so he claims.

As against this, Bernard Linsky claims that while Russell, in treating denoting phrases as incomplete symbols, denies that descriptions contribute anything to the logical forms of the sentences in which they occur, Neale (1990) holds that descriptions *do* contribute to logical form (Linsky 1992: 681). If so, it would seem that Neale does not honour Russell’s fundamental principle of *OD*, that denoting phrases are incomplete symbols. Now in a certain sense, Linsky may be correct that on Neale’s view descriptions *do* contribute to logical form; but given the very different conceptions of logical form employed by Neale and Russell, it is not clear that this could be used as the basis for a compelling objection that Neale fails to treat descriptions as incomplete symbols.⁴⁵ Neale provides an example in demonstration of the point. Consider the sentence ‘the king likes Russell’. The theory of descriptions, cast in terms of restricted quantifiers, predicts the following logical form:

$$(5_N) \quad [\text{the king } x](x \text{ likes Russell})$$

Neale writes:

⁴⁴ Neale’s position is undoubtedly better suited to the semantics of natural language than is Russell’s official position. One may, of course, take an entirely un-Russellian view of the role of definite descriptions in natural language, rejecting both Neale’s and Russell’s positions.

⁴⁵ I am inclined to think that Linsky’s discussion of the relation between Neale’s conception of logical form and the Russellian notion of *proposition* is quite good. Given the general tenor of Neale’s response to it however, he may not agree (Neale 1993: 90-2).

The variable “ x ” occupies the “subject position” of the formula “(x likes Russell)” and, so to speak, marks the position upon which the quantifier operates, the position that, in effect, represents the spot the quantifier occupies in surface syntax. [. . .] Now there is a sense in which the *variable* in (5_N) might be thought of as *complete-with-respect-to-a-sequence* by virtue of standing for an object in its own relativized way [. . .]. But however you look at it, the quantifier “[the king x]” that binds the variable is an incomplete symbol. It doesn’t even purport to stand for an object, not even when relativized to a sequence. (Neale 1993: 92-3)⁴⁶

As a response to Linsky’s charge, this is compelling. But it will be of no use in response to the objection I have raised. I have argued that Neale fails to honour Russell’s commitment to the fundamental principle of *OD* by failing to recognise that, in a sentence of the form ‘det F is G ’, it is not only the denoting phrase that lacks a propositional complement. For Russell, sentences of the forms ‘det F is G ’ and ‘ a is G ’ (taking ‘ a ’ as a genuine singular term) express propositions having radically different forms: in particular, they do not both predicate G -ness. Neale’s interpretation of the theory does not reflect this view.⁴⁷

Neale, it seems to me, runs together interpretive and decompositional analysis in a way that the Russell of *OD* does not. Neale’s analysis *has* to be (to some extent) decompositional *in the first instance* precisely because—unlike Russell—his project is that of compositional semantics: he has to take a sentence of natural language, break it down to its constituent expressions, and then show how the meaning of the whole can be recovered from the meaning of its parts and the way they are arranged. In relation to his project, it is entirely appropriate to enquire after the denotation of such descriptions as he ponders in the puzzle discussed above (i.e. ‘the way “the ϕ ” bears on propositional content’, and so on). There is a puzzle here *for Neale*—one that his use of restricted quantifiers solves quite neatly. But there is, as we saw, no puzzle for Russell. Why? Because what he aims to provide is not an account of *the way descriptions bear on propositional content*, but an SR-interpretive analysis of any sentential context in which a denoting phrase occurs. As we shall see in Chapter Five, it is important for

⁴⁶ I have made very minor alterations to Neale’s notation and numbering.

⁴⁷ In the preface to *Descriptions* (1990: ix) Neale claims to be defending the central theses of *OD*. I claim that the central theses of *OD* include a commitment to the primacy of SR-interpretive analysis. Neale does not, I claim, defend this aspect of Russell view (and nor should he, given his purposes).

Russell's purposes that his theory works in just the way it does: the modified theory at work in philosophy of language would not be suitable for these purposes.

5. The Three Logical Puzzles and the Notion of Scope

In *OD* Russell sets out three logical puzzles which an adequate theory of denoting will solve (*OD*: 47-8). It is sometimes thought that the puzzles are intended as either evidence in favour of Russell's view or part of an objection to rival theories. As against the first disjunct it should be noted that the puzzles merely present a condition of adequacy for theories of denoting: a theory that does not solve them is not fit for purpose. But the fact that a theory *does* solve them is not, in itself, evidence that that theory is to be endorsed. So the evidence in favour of Russell's view is not simply that it solves the puzzles—Frege's theory does this too—but is rather:

derived from the difficulties which seem unavoidable if we regard denoting phrases as standing for genuine constituents of the propositions in whose verbal expressions they occur. (*OD*: 45)

That is, the evidence for Russell's theory is that those theories based upon the denial of the fundamental principle of *OD* face unavoidable difficulties. Meinong's, he will argue, leads straightforwardly to contradiction; while Frege's—and so by extension the theory of denoting concepts and the theory of meaning and denotation—leads to the 'inextricable tangle' of the GEA. The theory of descriptions wins by default as it were, as the last man standing.

If the main argument against Frege is the GEA, then the role of the puzzles in *OD* becomes puzzling in itself. Being placed immediately after the initial criticism of Frege's view (*OD*: 46-7), they seem to be part of the objection to Frege (and possibly Meinong too)—which is what the second of the two disjuncts above attests. However, the paragraph that introduces them (beginning: 'A logical theory may be tested...' (*OD*: 47)) makes no mention of Frege, and gives no indication that they form part of an objection to anybody's view. This is, then, rather strange, if the puzzles form part of an objection to Frege.

It could be that all criticism of Frege's view stops where the puzzles begin. This is the view of Michael Kremer (1994), who uses the positioning of the puzzles to motivate the claim that Frege is not targeted by the GEA. But Kremer's view is extremely implausible, for textual evidence makes it overwhelmingly likely that the GEA targets both Frege and the theory of denoting concepts.⁴⁸ The correct interpretation is, I think, that the puzzles are not explicitly intended to form part of an objection to Frege. Admittedly this makes the placement of the puzzles strange indeed, but we might speculate that there is a rather sad reason for this. It is likely that Russell wrote *OD* in just 12 days towards the end of July 1905. We know also that his close friend Theodore Llewelyn Davies—one of the first friends he made at Cambridge—died on the 25th in a freak accident.⁴⁹ No surprise then if the article is less well-structured than it could have been (and no surprise either that the GEA could stand a re-write or two).

So the puzzles are not, I think, explicitly intended as part of an objection to Frege, or anyone else. They are, rather, intended to present a condition of adequacy for theories of denoting: a way of testing such theories. I will discuss the initial objections to Meinong and Frege in §6. For now I set out the solution of the puzzles by the theory of descriptions.

The first puzzle concerns the substitution of co-referring expressions, though it is phrased in terms of the substitution of *entities* in *propositions* (rather than *expressions* in *sentences*) (*OD*: 47-8). If *a* and *b* are identical then whatever is true of one is true of the other; and substituting one for the other in a proposition should not alter the truth value of that proposition. Since George IV wished to know whether Scott was the author of *Waverley*, and since Scott was the author of *Waverley*, we seem to be able to infer that George IV wished to know whether Scott was Scott. But this is presumably false. George IV did not wish to know anything about Scott's self-identity. As Russell puts it, 'an interest in the law of identity can hardly be attributed to the first gentleman of Europe' (*OD*: 48).⁵⁰

⁴⁸ This is discussed more fully in Chapter Four.

⁴⁹ Urquhart 1994.

⁵⁰ Attributing an interest in the law of identity to *the first gentleman of Europe* is not, by Russell's standards, the same as attributing such an interest to George IV. This is nit-picking of course.

On Frege's view the puzzle is avoided because, in intensional contexts, only substitutions of expressions having the same meaning (*Sinn*) are guaranteed to be truth-preserving. 'Scott' and 'the author of *Waverley*' express different meanings (*Sinne*), hence the substitution of one for the other is truth-preserving only in extensional contexts. Or, to engage with Russell's statement of the puzzle, Frege denies that the propositional complements of 'Scott' and 'the author of *Waverley*' are identical; so Leibniz's Law does not come into it, and the puzzle cannot get off the ground. The theory of descriptions also denies that 'Scott' and 'the author of *Waverley*' have identical propositional complements—though unlike Frege's view, it denies that 'the author of *Waverley*' has a propositional complement at all. Again then, Leibniz's Law does not come into it, and the puzzle cannot get started.

Although the denial that 'Scott' and 'the author of *Waverley*' have the same propositional complement suffices to defuse the puzzle, one may be left unsatisfied. For surely there is a sense in which George IV, by virtue of wishing to know whether the author of *Waverley* was Scott, *did* wish to know whether Scott was identical to Scott. Admittedly he would not have raised the question in those terms; but, to take an example of Russell's, had he seen Scott from a distance George IV might well have wondered 'is *that* Scott?' (*OD*: 52).

Frege's response holds up here, for he can deny that 'Scott' has the same propositional complement as 'that' (or whatever referring expression is chosen). But Russell, for whom, co-referring expressions—such as 'Scott' and, in the example, 'that'—*do* have the same propositional complement, must find another way.⁵¹ The solution invokes the distinction between 'primary' and 'secondary' occurrences, or 'scope permutations'. According to Russell (*OD*: 52), sentence (6) is strictly ambiguous between two readings.

(6) George IV wished to know whether Scott is the author of *Waverley*.

It could be taken to mean something like:

⁵¹ Again I make the un-Russellian assumption that 'Scott' and 'that' (as used in the example envisaged) are genuine singular terms referring to Scott.

- (6a) One and only one man wrote *Waverley*: George IV wished to know whether that man was Scott.

But normally (or so Russell claims⁵²) we take it to mean something along these lines:

- (6b) George IV wished to know whether: one and only one man wrote *Waverley* and Scott was that man.

Sentence (6a) results from according the description in (6) what Russell calls a ‘primary occurrence’—‘wide scope’⁵³. On this reading, (6) is taken to have a subject-predicate grammatical form, ‘ $G(\text{the } W)$ ’, where ‘ G ’ is the predicate ‘George IV wished to know whether ξ is identical to Scott’, and ‘ W ’ the predicate ‘ ξ wrote *Waverley*’. We then apply SR-interpretive analysis, eliminating the description from this sentence (*OD*: 52), moving from the subject-predicate form ‘ $G(\text{the } W)$ ’ to the existential quantification (6a’):

$$(6a') \quad (\exists x)(Wx \ \& \ (y)(Wy \rightarrow y = x) \ \& \ Gx)$$

That is: there is an x such that x wrote *Waverley*, only x wrote *Waverley*, and George IV wished to know whether x is identical to Scott. Or as Russell puts it: ‘George IV wished to know, concerning the man who in fact wrote *Waverley*, whether he was Scott’ (*OD*: 52). This seems to capture the intuition that, in a certain sense, George IV *did* wish to know, of Scott, whether he was identical to Scott.

In (6b) on the other hand, the description from (6) has been accorded a ‘secondary occurrence’—‘narrow scope’⁵⁴. On this reading, (6) is taken to have the subject-predicate form, ‘ $G(s = \text{the } W)$ ’, where ‘ G ’ is the predicate ‘George IV wished to know whether ξ is true’, ‘ s ’ is Scott, and ‘ W ’ is the predicate ‘ ξ wrote *Waverley*’. We then apply the SR-interpretive analysis, eliminating the description from the subordinate sentence ‘ $s = \text{the } W$ ’, concluding that (6b) asserts that George IV wished to know

⁵² Russell must be wrong here. A Strawsonian view is far more plausible: we normally intend a version of (6a), though what occurs to the left of the colon is presupposed rather than asserted. See Strawson (1950).

⁵³ More precisely, *wider* scope than the verb of propositional attitude.

⁵⁴ More precisely, *narrower* scope than the verb of propositional attitude.

whether $(\exists x)(Wx \ \& \ (y)(Wy \rightarrow y = x) \ \& \ s = x)$ is true.⁵⁵ That is: ‘George IV wished to know whether one and only one man wrote *Waverley* and Scott was that man’ (OD: 52).

Scope permutations are also called upon in response to the second puzzle (stated at OD: 48). For any well-formed declarative sentence, either it or its negation is true (but not both). But consider the sentence ‘the present King of France is bald’. By the aforementioned principle, one might suppose that either it or ‘the present King of France is *not* bald’ must be true. Yet since there is neither a bald present King of France nor a hairy one, neither sentence is true.

The sentence ‘the present King of France is bald’ is analysed thus:⁵⁶

- (7) The present King of France is bald.
 $(\exists x)(Kx \ \& \ (y)(Ky \rightarrow y = x) \ \& \ Bx)$

But there are two ways of denying (7) according as the description is afforded wide or narrow scope relative to the negation. Giving it wide scope (a primary occurrence), the denial of (7) is:

- (¬7a) The present King of France is not-bald.
 $(\exists x)(Kx \ \& \ (y)(Ky \rightarrow y = x) \ \& \ \neg Bx)$

This is the reading that seems to give rise to the puzzle, for (¬7a) is false. But if the description is given narrow scope (a secondary occurrence), the denial of (7) is:

- (¬7b) It is not the case that: the present King of France is bald.
 $\neg(\exists x)(Kx \ \& \ (y)(Ky \rightarrow y = x) \ \& \ Bx)$

⁵⁵ A knowledge of the notation of elementary logic can no more be attributed to the first gentleman of Europe than can an interest in the law of identity. I merely intend to indicate the structure of the proposition he entertains, according to Russell’s theory.

⁵⁶ Taking ‘ Kx ’ as ‘ x is presently King of France’ and ‘ Bx ’ as ‘ x is bald’.

This sentence, which denies the existence of a bald present King of France, is true. Hence one of ‘the present King of France is bald’ or its denial is true *if* the denial is read as (\neg 7b). The puzzle is thus resolved. Notice, however, that a similar response is open to Frege. Maintaining that the definite description is a singular term, he can trade on the ambiguity between internal and external negation, denying either the predicate (yielding either the False or a truth-value gap) or the sentence (yielding the True).

The final puzzle, concerning denials of being (*OD*: 48), does not require the notion of scope. If there is no difference between *A* and *B*, then we may say ‘the difference between *A* and *B* does not subsist’. But if this sentence is true, its subject expression picks out nothing of which the predicate can be predicated. Meinong is prepared to bite the bullet and accord a kind of ontological status to the non-subsistent difference between *A* and *B*. This solution is to be avoided however (cf. §6 below). Russell’s solution is familiar. To affirm the subsistence of the *F* (i.e. the difference between *A* and *B*) is not to attribute some particular thing with a certain property (*subsistence*), but rather to assert that a certain propositional function is true for at least one value. Hence to deny the being of something is to assert that *no* value of a certain propositional function is true—that is, to express a proposition of the form ‘ $\neg(\exists x)(Fx \ \& \ (y)(Fy \rightarrow y = x))$ ’.⁵⁷ Again, a similar Fregean response is available. *Subsistence*, *existence*, and so on are not taken to be properties of objects, but of concepts (cf. Frege 2007: §53). The sentence ‘the difference between *A* and *B* does not subsist’ thus affirms that the concept *difference between A and B* is null.

Russell’s theory thus provides elegant solutions to the three puzzles.⁵⁸ But other theories—notably Frege’s—offer solutions too, and hence appear to have *something* going for them. Russell’s initial objections to rival views will be discussed in the following section. The major objection presented in *OD*—the GEA—must wait until Chapter Four.

⁵⁷ Notice how in his explanation of the solution Russell freely employs the loose use of ‘denotation’ discussed above (§2): ‘according to the meaning of denotation lately explained, “the difference between *A* and *B*” has a denotation when *A* and *B* differ, but not otherwise’ (*OD*: 53).

⁵⁸ It should not be supposed that Russell’s solutions to the puzzles are necessarily the best available. My intention has been to set out Russell’s solutions, not to provide detailed assessment of them.

6. The Central Question of ‘On Denoting’

OD offers a theory of denoting. Such a theory will explain how it is that a sentence containing a denoting phrase comes to be about whatever it is about (understanding ‘about’ in the broad sense). As such we may frame the central question of *OD* thus:

- Central Question of *OD*: How is it that sentences containing denoting phrases come to be about whatever it is that they are about?

OD offers the following response. Taking the ‘the teacher of Plato is wise’ as an example, the Central Question asks how this sentence comes to be about Socrates. Russell’s answer is that ‘the teacher of Plato is wise’ is about Socrates in virtue of containing a denoting phrase that denotes him. The explanation of this use of ‘denotes’ is as follows:

if ‘*C*’ is a denoting phrase, it may happen that there is one entity *x* (there cannot be more than one) for which the proposition ‘*x* is identical with *C*’ is true [. . .]. We may then say that the entity *x* is the denotation of the phrase ‘*C*’. Thus Scott is the denotation of ‘the author of *Waverley*’. (*OD*: 51, emphasis added)

To say that ‘the teacher of Plato’ denotes Socrates is to say that the proposition expressed by (8) is true.⁵⁹

$$(8) \quad (\exists x)(Txp \ \& \ (y)(Typ \rightarrow y = x) \ \& \ x = s)$$

Thus ‘the teacher of Plato is wise’ is about Socrates because the denoting phrase occurring therein denotes him.

Crucially, Russell’s answer to the Central Question relies upon the fundamental principle of *OD*: ‘the teacher of Plato’ denotes Socrates, but neither Socrates nor anything else is its propositional complement. The theories to which Russell objects in *OD* all rely upon the denial of the fundamental principle. His strategy is thus to provide what amounts to an extended *reductio* of this denial, i.e. a *reductio* of the claim that every denoting phrase has a propositional complement of some kind.

⁵⁹ Taking ‘*Txy*’ as ‘*x* taught *y*’, ‘*p*’ as ‘Plato’, and ‘*s*’ as ‘Socrates’.

The simplest such view is that the propositional complement of a description is the entity which, uniquely, satisfies its descriptive condition. A version of this position is attributed to Meinong, who endorses ‘the principle of the indifference of pure Objects to being’ (Meinong 1904: 86). Russell objects to Meinong as follows:

[Meinong’s] theory regards any grammatically correct denoting phrase as standing for an *object*. Thus ‘the present King of France’, ‘the round square’ etc., are supposed to be genuine objects. It is admitted that such objects do not *subsist*, but nevertheless they are supposed to be objects. This is in itself a difficult view; but the chief objection is that such objects, admittedly, are apt to infringe the law of contradiction. It is contended, for example, that the existent present King of France exists, and also does not exist; that the round square is round, and also not round, etc. But this is intolerable; and if any theory can be found to avoid this result, it is surely to be preferred. (*OD*: 45)

This is the core objection to Meinong.⁶⁰ It is straightforward but has tended to be somewhat misunderstood.⁶¹

Russell describes the view that the round square does not (*even*) subsist as ‘difficult in itself’. He does not stop to explain, but no explanation is required: it *just is* difficult to make sense of something that neither exists nor subsists. This is not an objection so much as an observation. The objection is that, regardless of their ontological status, such objects generate contradictions. If *being square* implies not-*being round*, then the round square is both round and not-round; the existent present King of France exists but does not exist, and so on. Meinong granted all of this, but denied any contravention of the law of non-contradiction: such objects he held to be beyond its jurisdiction. Russell recognised this aspect of his view⁶², but deemed it ‘intolerable’ (*OD*: 45). Meinong’s view is thus dismissed as wildly implausible.⁶³

⁶⁰ Others have been extracted from *OD* (Farrell Smith (1985; 2005) notes five, however it strikes me that one has to really *want* to see them before they appear).

⁶¹ E.g. Sainsbury (1979: 102-3) goes wrong by attributing to Russell a misunderstanding of Meinong’s position.

⁶² Cf. ‘such objects, *admittedly*, are apt to infringe the law of contradiction’ (*OD*: 45 emphasis added).

⁶³ Farrell Smith (2005: 147) argues that the objection in *OD*, being based on the law of contradiction, is distinctively logical, and thus independent of the later objections based on Russell’s famously robust sense of reality. This interpretation is lent credence by casting the objection as a dispute concerning the scope of the law of contradiction: Russell is a universalist, Meinong is not. But even so, the objection is that curtailing the law’s jurisdiction is *intolerable*. What makes it intolerable is surely that it conflicts with

Frege's theory differs from Meinong's in providing, as the propositional complement of a denoting phrase, not the entity satisfying the relevant descriptive condition, but a *meaning* (a *Sinn*: I will follow Russell's terminology). Thus the sentence 'the King of England is bald' expresses a proposition containing the complex meaning <the King of England> and is about Edward VII in virtue of the logical relation holding between the former and the latter. On the present view, the proposition expressed by 'the King of France is bald' ought to be about the entity denoted by the complex meaning <the King of France>. There is, however, no such denotation: what, then, is the proposition about? If it is about nothing at all, then surely the (pseudo-) sentence must be nonsensical (for it predicates baldness of... nothing at all). But, says Russell, it is certain that the sentence is not nonsense, 'since it is plainly false' (*OD*: 46).

This claim is famously challenged by Strawson (1950), who maintains that, faced with a sincere utterance of 'the King of France is bald', one would not reply 'that's untrue', and, if pushed, would probably decline to agree *or* disagree with the statement. Strawson's contention is sound, but since the Central Question of *OD* does not concern the responses likely to be elicited by utterances of sentences containing denoting phrases, it is unclear what relevance it has here.⁶⁴ In any case, even if we tolerate a truth-value gap in the case of 'the King of France is bald', we surely cannot in Russell's next example:

consider such a proposition as the following: 'If *u* is a class which has only one member, then that one member is a member of *u*', or as we may state it, 'If *u* is a unit class, *the u* is a *u*'. This proposition ought to be *always* true. But 'the *u*' is a denoting phrase, and it is the denotation, not the meaning, that is said to be a *u*. Now if *u* is *not* a unit class, 'the *u*' seems to denote nothing; hence our

logical common sense, that it involves precisely the same 'failure of that feeling for reality which ought to be preserved even in the most abstract studies' (*IMP*: 169) to which Russell was later to appeal.

⁶⁴ I do not deny that Strawson's point is relevant to the question of the role of the theory of descriptions in the philosophy of language. It is sometimes suggested that the Russell of *OD* *must* have been concerned (to some extent) with natural language uses of descriptions, else he would not have responded so emphatically, in *MSOR*, to Strawson's criticisms. It should be noted however that in *MSOR* Russell points out the differences between his approach and Strawson's (e.g. 'My theory of descriptions was never intended as an analysis of the state of mind of those who utter sentences containing descriptions' (*MSOR*: 243)).

proposition would seem to become nonsense as soon as *u* is not a unit class.
(*OD*: 46)⁶⁵

He thus draws the conclusion that:

we must either provide a denotation in cases in which it is at first sight absent, or we must abandon the view that the denotation is what is concerned in propositions which contain denoting phrases. (*OD*: 47)

Russell himself endorses a version of the second disjunct. When a sentence contains an empty description, he will maintain that there is no entity that the proposition expressed is about. The Fregean response is to stipulate that all empty descriptions denote the null-class, and that, in general, an improper definite description denotes the class of objects satisfying the descriptive condition.⁶⁶ Therefore in the example given, if *u* is not a unit class, but has more than one member, the sentence ‘the *u* is a *u*’ is not meaningless but false. This fixes the logical worry: every well-formed sentence containing a denoting phrase is assured a truth-value even if the denoting phrase is improper.⁶⁷ Frege’s treatment is certainly neat and tidy, yet Russell is surely right to object that ‘though it may not lead to actual logical error, [it] is plainly artificial, and does not give an exact analysis of the matter’ (*OD*: 47).

The charge here is certainly not decisive but, in conjunction with the presentation of the theory of descriptions, it presents a strong case. The objection, up to this point, is that Frege’s answer to the Central Question of *OD* involves a certain amount of artificiality. In the GEA, however, Russell will make a far stronger claim: any theory whose answer to the Central Question invokes the distinction between, and logical relation of, the meaning and denotation of a denoting phrase is demonstrably incoherent. We turn now to that argument.

⁶⁵ One might object here that Russell equivocates over the use of ‘*u*’, using it differently in ‘if *u* is a class’ and ‘the *u* is a *u*’

⁶⁶ Cf. Frege 1964: §11; but compare his earlier treatment of ‘the greatest proper fraction’ (Frege 2007: §74n.)

⁶⁷ Notice however that on Frege’s view the sentence in question is false, not true as one might suppose it should be.

4. The Gray's *Elegy* Argument

As the aim of this chapter is to provide an interpretation of the notorious 'Gray's Elegy Argument' (GEA), it may be helpful to quote the argument, in its entirety, at the outset. The eight paragraphs are labelled 'A' to 'H', following the now standard practice (introduced by Blackburn & Code (1978)).

The Gray's *Elegy* Argument (OD: 48-51)

- (A) The relation of the meaning to the denotation involves certain rather curious difficulties, which seem in themselves sufficient to prove that the theory which leads to such difficulties must be wrong.
- (B) When we wish to speak about the *meaning* of a denoting phrase, as opposed to its *denotation*, the natural mode of doing so is by inverted commas. Thus we say:

The centre of mass of the solar system is a point, not a denoting complex;
 'The centre of mass of the solar system' is a denoting complex, not a point.

Or again,

The first line of Gray's *Elegy* states a proposition.
 'The first line of Gray's *Elegy*' does not state a proposition.

Thus taking any denoting phrase, say *C*, we wish to consider the relation between *C* and '*C*', where the difference of the two is of the kind exemplified in the above two instances.

- (C) We say, to begin with, that when *C* occurs it is the *denotation* that we are speaking about; but when '*C*' occurs, it is the *meaning*. Now the relation of meaning and denotation is not merely linguistic through the phrase: there must be a logical relation involved, which we express by saying that the meaning denotes the denotation. But the difficulty which confronts us is that we cannot succeed in *both* preserving the connexion of meaning and denotation *and* preventing them from being one and the same; also that the meaning cannot be got at except by means of denoting phrases. This happens as follows.
- (D) The one phrase *C* was to have both meaning and denotation. But if we speak of 'the meaning of *C*', that gives us the meaning (if any) of the denotation. 'The meaning of the first line of Gray's *Elegy*' is the same as 'The meaning of "The

curfew tolls the knell of parting day”,’ and is not the same as ‘The meaning of “the first line of Gray’s Elegy”.’ Thus in order to get the meaning we want, we must speak not of ‘the meaning of *C*’, but of ‘the meaning of “*C*”,’ which is the same as ‘*C*’ by itself. Similarly ‘the denotation of *C*’ does not mean the denotation we want, but means something which, if it denotes at all, denotes what is denoted by the denotation we want. For example, let ‘*C*’ be ‘the denoting complex occurring in the second of the above instances’. Then

C = ‘the first line of Gray’s Elegy’, and

the denotation of *C* = The curfew tolls the knell of parting day. But what we *meant* to have as the denotation was ‘the first line of Gray’s Elegy’. Thus we have failed to get what we wanted.

- (E) The difficulty in speaking of the meaning of a denoting complex may be stated thus: The moment we put the complex in a proposition, the proposition is about the denotation; and if we make a proposition in which the subject is ‘the meaning of *C*’, then the subject is the meaning (if any) of the denotation, which was not intended. This leads us to say that, when we distinguish meaning and denotation, we must be dealing with the meaning: the meaning has denotation and is a complex, and there is not something other than the meaning, which can be called the complex, and be said to *have* both meaning and denotation. The right phrase, on the view in question, is that some meanings have denotations.

- (F) But this only makes our difficulty in speaking of meanings more evident. For suppose *C* is our complex; then we are to say that *C* is the meaning of the complex. Nevertheless, whenever *C* occurs without inverted commas, what is said is not true of the meaning, but only of the denotation, as when we say: The centre of mass of the solar system is a point. Thus to speak of *C* itself, i.e., to make a proposition about the meaning, our subject must not be *C*, but something which denotes *C*. Thus ‘*C*’, which is what we use when we want to speak of the meaning, must be not the meaning, but something which denotes the meaning. And *C* must not be a constituent of this complex (as it is of ‘the meaning of *C*’); for if *C* occurs in the complex, it will be its denotation, not its meaning, that will occur, and there is no backward road from denotations to meanings, because every object can be denoted by an infinite number of different denoting phrases.

- (G) Thus it would seem that ‘*C*’ and *C* are different entities, such that ‘*C*’ denotes *C*; but this cannot be an explanation, because the relation of ‘*C*’ to *C* remains wholly mysterious; and where are we to find the denoting complex ‘*C*’ which is to denote *C*? Moreover, when *C* occurs in a proposition, it is not *only* the denotation that occurs (as we shall see in the next paragraph); yet, on the view in question, *C* is only the denotation, the meaning being wholly relegated to ‘*C*’.

This is an inextricable tangle, and seems to prove that the whole distinction of meaning and denotation has been wrongly conceived.

- (H) That the meaning is relevant when a denoting phrase occurs in a proposition is formally proved by the puzzle about the author of *Waverley*. The proposition ‘Scott was the author of *Waverley*’ has a property not possessed by ‘Scott was Scott’, namely the property that George IV wished to know whether it was true. Thus the two are not identical propositions; hence the meaning of ‘the author of *Waverley*’ must be relevant as well as the denotation, if we adhere to the point of view to which this distinction belongs. Yet, as we have just seen, so long as we adhere to this point of view, we are compelled to hold that only the denotation can be relevant. Thus the point of view in question must be abandoned.

1. Introduction

In *PoM* Russell had identified a class of propositional functions for which variation of argument was not independent of variation of function:

In general, ϕx is itself a function of two variables, ϕ and x ; of these, either may be given a constant value, and either may be varied without reference to the other. But in the type of propositional function we are considering in this Chapter, the argument itself is a function of the propositional function: instead of ϕx we have $\phi\{f(\phi)\}$, where $f(\phi)$ is defined as a function of ϕ . Thus when ϕ is varied, the argument of which ϕ is asserted is varied too. (*PoM*: §103, 104)

Such propositional functions he called ‘quadratic forms’.

These forms, and the relations between the variables occurring therein, were clearly still central to Russell’s attempts to respond to the paradoxes in 1905. In *OF*, still endorsing the theory of meaning and denotation, Russell writes:

It seems likely that meaning-variation must be distinguished from entity-variation, and that two variables of which one *means* [i.e. occurs in meaning-position] and the other *is* [i.e. occurs in entity position] can only be equal *by accident*, and can’t be kept equal throughout variation. (*OF*: 360, emphasis added)

In the quadratic form $\phi\{f(\phi)\}$ for instance, ϕ occurs in a meaning-position in its leftmost occurrence, but in an entity-position in its rightmost occurrence. It becomes

imperative, then, to attempt to understand the connection between these two modes of occurrence. On this point Russell recognises that:

if we assert a connection between a variable in a meaning-position and a variable in an entity-position, we must avoid denoting complexes, since these will stand for their meaning in the one position and for their denotation in the other. (*OF*: 361)

And since variables have both meaning and denotation, the connection between the two occurrences of ϕ in $\phi\{f(\phi)\}$ is unclear.

A further problem concerns the symbolism. If ' $\phi(x)$ ' represents the application of the value of the variable ' ϕ ' to the value of ' x ', then one might assume that ' $\phi(\phi)$ ' represents the application of the value of ' ϕ ' to itself. However since the structure of ' $\phi(\phi)$ ' is:¹

- $_m[\phi]_m/_e[\phi]_e$

the variable has a different value in the different positions (or at least if it has the same value this is only 'by accident'). To symbolise the intended complex the notation must be altered from ' $\phi(\phi)$ ' to something else. We might employ the subscript-plus-bracket notation introduced in Chapter Two, or something slightly simpler, such as a circumflex or inverted commas. Thus Russell:

When we write

$$\vdash : p \supset q . \supset q : \equiv : q \supset p . \supset . p$$

we state that the equivalence in question holds for *any* value. When we wish to speak of the function itself, *i.e.* the constant meaning, we write $\hat{p} \supset \hat{q} . \supset . \hat{q}$ instead of $p \supset q . \supset . q$. [...] The circumflex has the same sort of effect as inverted commas have. E.g. we say

Any man is a biped;

¹ The notation here is introduced in Chapter Two, §4.2. To recapitulate: the bracket-plus-subscript notation (e.g. ' $_m[]$ ' and ' $]_m$ ', and ' $_e[]$ ' and ' $]_e$ ') distinguishes the mode of occurrence of the term occurring within the brackets. The slash '/' simply separates variables to keep things neat.

“Any man” is a denoting concept.

The difference between $p \supset q . \supset . q$ and $\hat{p} \supset \hat{q} . \supset . \hat{q}$ corresponds to the difference between any man and “any man”. (FN: 128-29)

This addition to the symbolism appears to be innocent; but it must be ensured that we understand precisely what is represented at the propositional level by the circumflex or inverted comma notation.

We have two issues, then—the connection between meaning- and entity-occurrence, and the inverted comma notation. Russell’s investigation of their interaction with the theory of meaning and denotation forms the backdrop to the Gray’s *Elegy* Argument (GEA) which, though it occurs in *OD*, is first formulated in *OF*, just lines before the discovery of the theory of descriptions.²

I hold that the GEA constitutes a decisive objection to Russell’s earlier view: the role of the present chapter is simply to lay the argument out in full. I begin in §2 by identifying the targets of the GEA. I claim that the argument targets both Russell’s earlier theory of denoting and Frege’s distinction between *Sinn* and *Bedeutung*. The two theories share, I will claim, a common assumption in virtue of which both are targeted. I shall argue against the claim that the common assumption is the thesis that definite descriptions are singular terms.³ Russell was not, I shall argue, committed to this thesis at all. Rather, what the earlier Russell and Frege shared—at least in Russell’s eyes—was the view that the answer to Central Question of *OD* (namely: How is it that a sentence containing a denoting phrase comes to be about whatever it is about?) should appeal to the relation between meaning and denotation.

In §3 and §4 I present an original interpretation of the GEA. In §3 I give a brief overview of the course that the argument will take. In §4 I offer a fuller interpretation, including detailed analysis of the text. Finally, in §5 I briefly discuss the implications of the argument for Frege’s position. The present chapter aims only at setting out the GEA.

² *OD*: 48-51; *OF*: §§35-9. When quoting from the GEA I will give the letter of the paragraph after the page reference, e.g. ‘*OD*: 50 G’.

³ This view is advocated by Nathan Salmon (2005) and, indirectly, Peter Hylton (1990).

The demonstration of its connection with the discovery of the theory of descriptions and Russell's attempts to solve the paradoxes must wait until Chapter Five.

2. The Targets of the Gray's *Elegy* Argument

Early commentaries⁴ rather took it for granted that the GEA targets Frege. There is no surprise in this, as the textual evidence from *OD* seems clearly to implicate Frege as the target. But since Russell claims that Frege's theory is 'very nearly the same' (*OD*: 42n.) as his own earlier theory of denoting, it is likely, going solely on the textual evidence from *OD*, that any objection to Frege is *also* an objection to that theory.

In its initial pre-publication formulation (in *OF*) the objection targeted Russell's earlier theory. It is probably for this reason that commentators have not struggled to identify, in the published version, underlying assumptions which Frege did not share.⁵ In a highly influential paper, P. T. Geach, commenting on Searle's (1958) interpretation, refocused attention on Russell's earlier view, stating that 'readers of [*OD*] will find it best simply to ignore [Russell's] use of Frege's name' (Geach 1959: 72).⁶ I commend the refocusing of attention upon Russell's earlier view, but it cannot be correct to dismiss Frege in this way: the textual evidence of *OD* is overwhelming in this regard (cf. §3.1 below).⁷

In the following subsection I will briefly set out the textual evidence supporting the claim that the GEA targets both Frege's theory and Russell's earlier view. Since, as I will eventually argue (§2.3), Russell is wrong to claim that his earlier theory is 'very nearly the same' (*OD*: 42n.) as Frege's, we must try to discover why he took them to be so. It could be that Russell simply misunderstood Frege's position. However it is both more charitable and more fruitful to suppose that, despite the significant differences between the two positions, there is some common assumption that they share. In §2.2 I will discuss the claim that the aim of the GEA was 'to supplant the view that *a definite description is a singular term*' (Salmon 2005: 1076, emphasis added), a view which is

⁴ E.g. Jones 1910, Church 1943, Butler 1954 and Searle 1958.

⁵ See, for example, Levine's (2004) careful examination of the differences in the underlying epistemologies of Frege and Russell.

⁶ Butler 1954 also contains an early discussion of Russell's position in *PoM*, though he still sees the GEA as targeting Frege.

⁷ There is obviously no justification for ignoring Frege's name when Russell—outside the context of the GEA—clearly intends to discuss Frege (e.g. *OD*: 45-48). And there is actually no cause for ignoring Frege's name as it occurs in the GEA passages, since it doesn't there occur at all.

commonly attributed to both Frege and the pre-*OD* Russell. I argue that Russell was not committed to the view, and, in §2.3, suggest an alternative shared assumption.

2.1. Textual evidence

Strong textual evidence supports the view that the GEA targets *both* Frege and the earlier Russell. Firstly, Russell writes:

I have discussed this subject [i.e. denoting] in [*PoM*], Chap. V, and §476. The theory there advocated is very nearly the same as Frege's, and is quite different from the theory to be advocated in what follows. (*OD*: 42n.)

Thus anyone holding that the argument targets only one of Frege's or Russell's earlier theory owes an explanation of why it does not also target the other, given that the two are, to Russell's mind, so similar.

Secondly, although the GEA contains no explicit indication of its target, it is surely significant that the argument is framed in terms of 'meaning' and 'denotation'. Plausibly, the GEA's use of 'meaning' and 'denotation' is continuous with their use elsewhere in *OD*. In particular, it is presumably continuous with the introduction of those terms in connection with Frege (*OD*: 45). If this suggests that the argument targets Frege's theory, then in the absence of any argument to the contrary, and given the first point, it suggests that the argument also targets Russell's earlier theory.

Alternatively we may argue in the opposite direction. Given that the GEA is framed in terms of 'meaning' and 'denotation', it is reasonable to suppose that those terms are used with the same sense that they had in the original formulation of the argument in *OF*. After all, certain passages of the *OF* formulation appear verbatim in *OD*. If this suggests that the argument targets Russell's earlier view, then given its perceived similarity to Frege's theory, and in the absence of any argument to the contrary, it suggests that the argument also targets Frege.

This evidence—in conjunction with the complete lack of textual support for its denial—is overwhelming. So strong is it, that even if it were shown that the GEA failed to engage in any substantive way with Frege's theory, it would be more plausible to

maintain that Russell had misunderstood Frege's position, than that he had not intended⁸ the argument to target it.

2.2. *Definite descriptions as singular terms?*

Nathan Salmon (2005: 1076) claims that the suggestion that the GEA targets *just* Frege, or *just* the earlier Russell, or *just* both, is wrong: what is targeted is a certain thesis, common to far more philosophers than just Frege and the earlier Russell:

Russell's ultimate aim in [OD] is to supplant the view that a definite description is a singular term. This view is by no means particular to Frege or the earlier Russell. It was also held, for example, by John Stuart Mill and Meinong. And it remains commonplace among language scholars today. [. . .] The burden of [OD] is to depose this very basic, and seemingly innocuous, account of definite descriptions. (Salmon 2005: 1076)

It is exactly this basic, and seemingly innocuous, account—nothing less—that I believe Russell is ultimately attempting to refute in his 'Gray's *Elegy*' argument. [. . .] He thus intends to overthrow by his argument *both* Frege and his former self. But not only these two. [. . .] [T]he 'Gray's *Elegy*' argument effectively aims to debunk Mill, Frege, Meinong, and every other philosopher of language to have come down the pike—including the author of [PoM]. (Salmon 2005: 1077-78)

Now Frege certainly treated definite descriptions as singular terms; but I will argue that the Russell of *PoM* did not (rather, definite descriptions in *PoM* involve the notion of *denoting* and hence of *generality*). Consequently, to the extent that the assumption that he did so take definite descriptions informs an interpretation of the GEA, that interpretation is suspect.

Quite how widespread this understanding of Russell's position in *PoM* is, is difficult to judge (it is not often discussed—perhaps because it is widely assumed). It is explicitly attributed by Salmon (as above) and Pelletier & Linsky (2009: 40), and is

⁸ One might claim that since the GEA was developed in *OF*, in isolation from any discussion of Frege, the argument was not *intended* to target Frege, but was merely understood by Russell as *having an application to* Frege's position. Maybe so, but this point has to do with the semantics of 'intention' and 'intends': the exegetical point stands independently of such matters.

almost explicit in Blackburn & Code 1978 (68) and Levine 2005.⁹ I shall argue that it is indirectly attributed to the early Russell in Peter Hylton's important work *Russell, Idealism and the Emergence of Analytic Philosophy* (1990). Hylton does not explicitly address the question of descriptions as singular terms in *PoM*, but he does attribute to the Russell of *PoM* a related claim, *the principle of truth-value dependence*:

Principle of Truth-Value Dependence (TV Dep)

If *p* is a proposition containing a denoting concept, the truth-value of *p* is 'dependent upon the truth-value of the proposition obtained from [*p*] by replacing the denoting concept by the denoted entity' (Hylton 1990: 251).

I argue (§2.2.1) that to attribute (TV Dep) to Russell (at least as far as definite descriptions are concerned) is tantamount to attributing to him the thesis that definite descriptions are singular terms. Of course much depends on how we understand 'singular term', and part of my task is to settle on an appropriate understanding. I shall argue in this way: if the Russell of *PoM* was committed to (TV Dep), then he was also committed to the thesis that definite descriptions are singular terms; but since he was not committed to the latter (§2.2.2), he was not committed to the former. Salmon and Hylton have, in that case, mischaracterised Russell's position in *PoM*. I will argue (§2.2.3) that what is most significant about *OD* is not, *pace* Salmon, that Russell no longer treats descriptions as singular terms, but rather that he no longer treats denoting phrases as having 'meaning in isolation'—in other words that he now endorses what in Chapter Three I called 'the fundamental principle of *OD*', namely that denoting phrases are incomplete symbols.

2.2.1. Singular terms and the principle of truth-value dependence

Hylton introduces (TV Dep) as follows:

The crucial idea for understanding denoting is that a proposition may be *about* an object which it does not contain. It is by no means obvious how to make sense of this idea within the context of [the general framework of *PoM*]. [. . .]

⁹ One might claim that it is in a certain sense *implied* by introductory texts in philosophy of language which (implicitly perhaps) represent the Russell of *OD* as opposing Frege on the question of whether definite descriptions (and proper names) are genuine singular terms. Witness, for instance: 'Russell was exercised in *OD* by what he took to be a number of serious defects in Proper-Name treatments of descriptions' (McCulloch 1989: 41).

[A] natural way to do so is to say that for a proposition containing a denoting concept to be about some other entity is for the truth-value of that proposition to be dependent upon the truth-value of the proposition obtained from it by replacing the denoting concept by the denoted entity. (Hylton 1990: 251)

Replacing the denoting concept in ⟨1⟩ with the denoted entity, we arrive at ⟨2⟩:

- ⟨1⟩ ⟨<the teacher of Plato> is wise⟩
- ⟨2⟩ ⟨Socrates is wise⟩

Hylton clearly intends (TV Dep) to posit a stronger connection between the truth-values of ⟨1⟩ and ⟨2⟩ than mere co-variance. The principle says that the truth-value of ⟨1⟩ *depends upon* that of ⟨2⟩. Thus the truth of ⟨1⟩ must be grounded in the facts that <the teacher of Plato> denotes Socrates, and that the result of replacing the former with the latter (in ⟨1⟩) is a true proposition. On this view, ⟨1⟩ attributes wisdom to some particular entity and is true just in case that entity is wise. That entity happens to be Socrates (since <the teacher of Plato> denotes him), so ⟨1⟩ is true just in case there is a true proposition attributing wisdom to Socrates—that is, ⟨1⟩ is true just in case ⟨2⟩ is.

This is, admittedly, a neat explanation of why ⟨1⟩ is about Socrates; and since the notion of *aboutness* in *PoM* (and in Russell's work in this period generally) is unclear, any explanation is welcome. One drawback is that although it explains why ⟨1⟩ is about Socrates, it does not explain why ⟨2⟩ is. It might seem that no explanation is required here: after all, what else *could* it be about? Well it could be about *wisdom* couldn't it? Russell thinks not (*PoM*: §48), though his failure to even acknowledge the possibility is unfortunate given its apparent viability.¹⁰ At root, the difficulty is that on Hylton's interpretation, the truth of a proposition containing a denoting concept is a matter of the truth-value of another proposition, while the truth of a singular proposition (i.e. one devoid of denoting concepts) has little or nothing to do with the truth of any other proposition. Thus Hylton attributes to Russell a hotchpotch theory of truth.

¹⁰ Cf. Ramsey 1931. See Gaskin 2008 for discussion and amplification.

Certainly Russell has nothing terribly clear to say about truth in *PoM*. Nonetheless, in *MTCA*, published only one year after *PoM*, while still endorsing the theory of denoting concepts, he advocates truth-primitivism:

some propositions are true and some false, just as some roses are red and some white. [. . .] What is truth, and what falsehood, we must merely apprehend, for both seem incapable of analysis. (*MTCA*: 75-76)¹¹

Hylton is well aware that his interpretation stands in opposition to Russell's truth-primitivism:

The idea of the truth or falsehood of one proposition depending upon that of another is clearly quite alien to [Russell's general position]. It amounts, indeed, to the introduction of something like the correspondence theory of truth for the special case of those propositions which contain denoting concepts: whether such a proposition is true depends upon whether there is a corresponding fact, where a fact is a true proposition which does not contain a denoting concept, or a combination of such propositions. (Hylton 1990: 209)

But he is willing to ascribe (TV Dep) to Russell, even in the face of this tension, as it seems to afford an explanation of various of Russell's moves in the GEA.

Firstly, it is suggested that at least part of the argumentation in the GEA is intended to demonstrate the impossibility of there being a proposition which is about a denoting concept in virtue of containing that concept. For instance, the proposition ⟨the teacher of Plato⟩ denotes Socrates) is apparently (or is intended to be) a true proposition about ⟨the teacher of Plato⟩; but if (TV Dep) holds, then its truth depends upon the truth of ⟨Socrates denotes Socrates⟩, which is obviously false:

[(TV Dep)] has the consequence that there are no true propositions which say that one entity denotes another; but clearly there must be such propositions if the theory of denoting concepts is correct. (Hylton 1990: 252)¹²

¹¹ For a careful discussion of Russell on truth see Korhonen 2009.

¹² One response to this problem is to distinguish different modes of occurrence. Perhaps the denoting concept in ⟨the teacher of Plato⟩ denotes Socrates) occurs *non-denotatively*? Hylton describes the attempt to follow through this idea (in *OF*) as 'showing Russell attempting to accommodate the failure of (TV Dep)' (1990: 253). This idea, stripped of any commitment to (TV Dep), looms large in my discussion of the GEA.

The second way in which (TV Dep) is supposedly implicated in the GEA concerns the final paragraph of the argument (*OD*: 50-51 H)¹³. There Russell implies that the fact that ‘Scott is the author of *Waverley*’ and ‘Scott is Scott’ express different propositions is problematic for his earlier view. But there is not, on the face of it, any obvious difficulty here for the theory of denoting concepts, which anyway distinguishes the two propositions. What then could Russell’s point be? We may answer this question, Hylton (1990: 253) says, ‘if we suppose that Russell is taking for granted something like [(TV Dep)]’. (TV Dep) has it that the truth-value of ⟨George IV wished to know whether Scott was <the author of *Waverley*>⟩ is dependent upon that of ⟨George IV wished to know whether Scott was Scott⟩. We are to suppose that the former is true, but since the latter is false, the former *cannot* be true, if we endorse (TV Dep).

In the above ways one can attempt to make sense of the GEA by associating the theory of denoting concepts with (TV Dep). This will all be to no avail, however, if it can be demonstrated that Russell did not endorse (TV Dep). But before demonstrating that (TV Dep) commits one to the thesis that definite descriptions are singular terms, let us settle on an understanding of ‘singular term’. A singular term is, according to Salmon (2005: 1072), an expression whose semantic function is to designate exactly one entity. But what exactly is *designation*?

Like *truth*, *designation* is connected to the notion of *aboutness*. To say that an expression designates an entity is to say that declarative sentences containing that expression express propositions that are *about* that entity.¹⁴ We have encountered two ways in which a proposition may be about an entity.¹⁵

(Ab1) A proposition may be about an entity *e* in virtue of containing *e*.

(Ab2) A proposition may be about an entity *e* in virtue of containing a denoting concept *d*, such that *d* denotes *e*.

Where definite descriptions are concerned, (Ab2) is in play.

¹³ To recapitulate: when citing a passage from the GEA I use ‘A’-‘H’ to indicate the relevant paragraph(s).

¹⁴ In this respect Salmon’s terminology is continuous with Russell’s in certain of his unpublished manuscripts, e.g. *OMD*.

¹⁵ Cf. Chapter One, §3.2.

Just as we can attribute a truth-value to a sentence based upon the truth-value of the proposition it expresses, so we can speak of a sentence's being *about* a certain entity based upon the associated proposition's being about that entity. So if 'the teacher of Plato' is a singular term designating Socrates, sentences of the form 'the teacher of Plato is *G*' express propositions which are about Socrates in the (Ab2) sense. And if Russell is committed to (TV Dep), then such sentences express propositions the truth-values of which are dependent upon the truth-value of propositions which are about Socrates in the (Ab1) sense.

We can cash out the distinctions in play here in more contemporary terms. According to the theory of descriptions, (1) expresses an *object-independent* proposition, whose form is more accurately represented by (1a):

- (1) The teacher of Plato is wise.
 (1a) $(\exists x)(Txp \ \& \ (y)(Txy \rightarrow y = x) \ \& \ Wx)$

While—assuming that 'Socrates' is a genuine singular term—(2) expresses an *object-dependent* proposition, whose form is more accurately represented by (2a):

- (2) Socrates is wise.
 (2a) Ws

Stephen Neale cashes out the distinction between object-dependent and object-independent propositions in these terms:

A genuine referring expression '*b*' may be combined with a (monadic) predicate expression to express an *object-dependent* thought¹⁶, a thought that simply could not be expressed or even entertained if the object referred to by '*b*' did not exist. A definite description 'the *F*', by contrast, although it *may* in fact be satisfied by a unique object *x*, can be combined with a (monadic) predicate to express a thought that is not contingent upon the existence of *x*. (Neale 1990: 5-6)

¹⁶ For present purposes read 'thought' as 'proposition'.

An object-dependent proposition will be true just in case the entity upon whose existence it is dependent has the property expressed by the predicate expression. The truth of an object-independent proposition will not depend upon any *particular* entity's having the property expressed by the predicate expression. There may, as a matter of fact, be exactly one entity satisfying the descriptive condition, but the proposition would subsist even if no entity (or more than one) satisfied the descriptive condition; for as we might put it, that entity does not enter into the truth-condition of the proposition expressed.

Returning to our example, the crucial difference between the propositions expressed by (1a) and (2a) is this: the state of each and every entity within the domain of the quantified variables is relevant to the truth-value of the proposition expressed by (1a), while the truth-value of the proposition expressed by (2a) is a matter only of the state of the entity designated by the singular term occurring therein. This is the distinction between object-dependent and object-independent propositions, and it squares up exactly to the distinction between singular terms and quantifier expressions (assuming that one follows Russell in taking genuine proper names to be singular terms and denoting phrases to be quantifier expressions). Let us then adopt the following loose characterisation of a singular term:

(ST) 'a' is a singular term just in case, when it occurs in a sentence *S* of the form '*a* is *G*', the truth-value of the proposition expressed by *S* is a matter of how things stand with the entity designated by 'a'.^{17 18}

Characterising singular terms in terms of object-dependent propositions involves a departure from Russell's terminology, but not from the spirit of the notions he employed. He writes for instance that if I assert 'I met a man' then:

the whole human race is involved in my assertion: if any man who ever existed or will exist had not existed or been going to exist, the purport of my proposition would have been different. (*PoM*: §62, 62)

¹⁷ This (ST) is not to be confused with Salmon's (2005: 1082).

¹⁸ To clarify: I intend the phrase 'is a matter of how things stand with...' in such a way that the truth of 'the teacher of Plato is wise'—according to the theory of descriptions—is *not* a matter of how things stand with Socrates, but rather a matter of how things stand with everything within the relevant domain.

This sets up a clear distinction between object-dependent propositions, in which the entity that the proposition is about actually occurs in the proposition, and object-independent propositions in which every entity in the relevant domain is relevant to the truth-value of the proposition. The distinction may only be implicit in *PoM*, and is certainly couched in different terms, but it is there nonetheless and recognisably so.

(ST) enables us to see exactly why commitment to (TV Dep) would commit Russell to the thesis that definite descriptions are singular terms. By (TV Dep), the truth of sentence (1) depends upon the truth of proposition $\langle 2 \rangle$. This is illustrated as follows. (1) is true (in the derivative sense of truth applicable to sentences) if it expresses a true proposition; (1) expresses $\langle 1 \rangle$; $\langle 1 \rangle$ is true, according to (TV Dep), just in case $\langle 2 \rangle$ is true; and the truth-value of $\langle 2 \rangle$ is a matter of how things stand with Socrates. ‘The teacher of Plato’ is, then, the kind of expression which, when it occurs in a sentence of the form ‘the teacher of Plato is *G*’, expresses a proposition whose truth-value is a matter of how things stand with Socrates (that is, the entity designated by ‘the teacher of Plato’). But to be an expression of that kind just is, according to (ST), what it is to be a singular term designating Socrates.

According to an alternative, but perhaps common, use of ‘singular term’, singular terms are simply to be contrasted with incomplete symbols. Now if to be a singular term is *just* to not be an incomplete symbol, then of course the Russell of *PoM* did take descriptions to be singular terms. But this conception of singular terms is so general as to be almost useless. For instance, suppose one takes it, as some do, that the semantic value of *OD*’s denoting phrases are second-level functions.¹⁹ Are we not then entitled to deem *OD*’s denoting phrases singular terms, since on this view they have second-level functions as their propositional complements? Under this proposal *all* quantifier expressions turn out to be singular terms, collapsing a distinction that ought to be preserved. What is wrong with the present understanding of singular terms is that it ignores the central issue, namely *aboutness*. Singular terms and quantifier expressions enable one to speak about the world in significantly different ways. A serviceable account of singular terms must therefore preserve the contrast with quantifier

¹⁹ E.g. Miller (1998).

expressions, but the proposal that to be a singular term is simply not to be an incomplete symbol fails in this regard. For this reason, when we ask whether definite descriptions are taken, by the Russell of *PoM*, to be singular terms, we are asking after (say) the way in which a sentence such as (1) is about Socrates. (ST) offers a characterisation of singular terms that preserves the contrast with quantifier expressions by appealing to the notion of designation, which is cashed out in terms of object-dependence. As I have urged, though this terminology is anachronistic, the underlying notions are not.

Let us pause, for a moment, to ask whether (ST) is the characterisation of singular terms that Salmon has in mind. One limitation of (ST) is that it says nothing of expressions which fail to designate. Salmon, on the other hand, writes:

an expression may have the semantic function of designating a single individual without necessarily fulfilling its function. Hence, ‘the present king of France’ is not disqualified [from being a singular term] simply because France is no longer a monarchy (and would not have been disqualified even if France had never been a monarchy). (2005: 1072n.)

Thus it seems that (ST) does not capture all that Salmon’s characterisation of singular terms captures. But arguably, whatever it is that (ST) leaves out can have no real bearing on a discussion of the views of the early Russell. To see why, let us consider sentence (3):

(3) The present king of France is wise.

If ‘the present king of France’ is a significant expression, as the Russell of *PoM* certainly held that it was, then (3) certainly expresses a proposition (call it ‘⟨3⟩’). Now there either is or is not some unique entity satisfying the descriptive condition *is presently king of France*. Let us consider the two possibilities.

(i). If there is no entity satisfying the descriptive condition, then the proposition must be false.²⁰ Notice the contrast here with the treatment of the arch singular term, the (genuine) proper name. Russell’s view is that there can be no such thing as a genuine proper name that lacks a bearer:

²⁰ It must be true or false—and it certainly isn’t true.

“*A* is not” must always be either false or meaningless. For if *A* were nothing, it could not be said not to be; “*A* is not” implies that there is a term [entity] *A* whose being is denied, and hence that *A* is. Thus unless “*A* is not” be an empty sound, it must be false—whatever *A* may be, it certainly is. (*PoM*: §427, 449)

For Russell the idea of a bearer-less proper name is incoherent. Thus if definite descriptions are singular terms, then insofar as they may be empty, they are singular terms of a different kind to proper names. This new kind of singular term would be one which tolerates the formation of sentences expressing propositions that do not conform to either of (Ab1) or (Ab2)—for there is no entity that ⟨3⟩ is about. Yet (Ab1) and (Ab2) are central to Russell’s framework of propositions in *PoM*. As such, the proposal that ‘the present king of France’ is a singular term designating nothing at all, and yet contributing to the formation of significant declarative sentences like (3), is highly implausible as an account of Russell position in *PoM*. Indeed it is difficult to see how it could even find an application to Russell’s position: that all propositions are about some entity (or entities) is near axiomatic for Russell in *PoM*, yet the present proposal tolerates exceptions.²¹

(ii). Let us then consider the second alternative, the possibility that there *is* some unique entity satisfying the descriptive condition *is presently king of France*. There is certainly no *existing* present king of France. But as is well-known, Russell countenanced entities which do not exist, but (merely²²) *subsist* or have *being* (*PoM*: §427, 449). Perhaps then, ‘the present king of France’ designates a non-existent being. ⟨3⟩ will then be taken as true or false depending upon how we evaluate propositions containing non-existent entities (we need not investigate this mode of evaluation: suffice it to note that ⟨3⟩ is, presumably, false). This second possibility, unlike the first, is in conformity with (Ab1) and (Ab2): ⟨3⟩ is about the non-existent present king of France in the (Ab2) sense.

Now I don’t, for the moment, want to deny that, as an interpretation of Russell, this view is an *option* (though I think it is the wrong one)²³. For now I simply note that if

²¹ A version of this proposal (as an interpretation of Russell) is considered and rejected below (§2.2.2.2).

²² The tendency to think of *being* as a ‘lesser’ ontological status than *existence* is not a good one.

²³ Cf. §2.2.2.1 below.

Salmon is attributing to Russell the view that definite descriptions are singular terms against *this* backdrop—and I should add that I do not think he is—so that empty definite descriptions are accounted for by positing non-existents, then (ST) does not, after all, miss out anything of importance. We were entertaining the possibility that (ST) was unfair to Salmon’s characterisation of singular terms on the grounds that it was too narrow, that it said nothing about empty definite descriptions. But now we have come full circle and *deny* that there are empty definite descriptions, by appealing to Russell’s distinction between *existence* and *being*. If there are no empty descriptions, (ST) is not too narrow after all. I take it then, that (ST) captures enough of what is important about Russell’s general position in *PoM* to be fruitfully applicable to that position, but is also not unfair to the notion of *singular term* at work in Salmon’s paper.

2.2.2. *Definite descriptions as singular terms in PoM*

We noted in the previous section that, for Russell in *PoM*, no proper name fails to indicate some entity; a proper name which indicates nothing is not, properly speaking, a proper name at all, but a meaningless sound. As far as *indication* goes, the same holds for denoting phrases: a denoting phrase that indicates no denoting concept is not, properly speaking, a *phrase* at all, but just another meaningless mark or sound. But Russell holds that *all* denoting phrases have an indication, namely a denoting concept. The question is, then, whether there can be denoting concepts that are empty, that denote nothing. Russell openly acknowledges this possibility:

It is necessary to realize, in the first place, that a concept may denote although it does not denote anything. This occurs when there are propositions in which the said concept occurs, and which are not about the said concept, but all such propositions are false. (*PoM*: §73, 73)

As such, there being no unique entity that is *F* is no barrier to there being a proposition expressed by a sentence of the form ‘the *F* is *G*’. But if the proposition is not about the denoting concept itself, and is not about the denotation (since, *ex hypothesi*, there is none), what could it be about?

If the Russell of *PoM* holds that definite descriptions are singular terms, then there are two plausible responses to the above question. First, one might renege on the claim that there is no denotation, declaring Russell’s statement to the contrary a slip. When

there is no unique F , ‘the F is G ’ will express a proposition which is about a non-existent entity. For instance, in §2.2.1 we entertained the suggestion that ‘the present king of France’ designates a non-existent entity: it *indicates* <the present king of France>, which *denotes* the non-existent present king of France. (3) is then significant in virtue of expressing the false proposition ⟨3⟩:

⟨3⟩ ⟨<the present king of France> is wise⟩

In §2.2.2.1 I indicate how recent developments in Russell scholarship tell against this proposal. If the Russell of *PoM* endorsed the thesis that definite descriptions are singular terms, it was certainly not against this backdrop.

The second, more promising, response to the question raised above involves maintaining the interpretation according to which Russell is committed to the thesis that definite descriptions are singular terms, but acknowledging and accommodating the fact that not all denoting phrases have a denotation. This is, no doubt, the position that Salmon and Hylton have in mind. In §2.2.2.2 I argue that even against this backdrop, proper consideration of the nature of denoting suggests that Russell should not be seen as committed to the thesis in question.

2.2.2.1. The ontology of PoM

Russell scholarship has seen, in recent years, a move away from the *Standard View* (as Griffin (1996) calls it) of the origins of the theory of descriptions, the view ‘that the theory of descriptions was intended primarily as a contribution to ontology, a device (as Quine [(1966: 5)] put it) for “dispensing with unwelcome objects”’ (Griffin 1996: 24). The Standard View has gone hand-in-hand with the claim that the ontology of *PoM* is (in some, most often pejorative, sense) ‘quasi-Meinongian’—that is ‘unrestrained’ (Quine 1966: 4) or ‘intolerably overcrowded’ (Ayer 1971: 28). Thus the move away from the Standard View has heralded a move away from the quasi-Meinongian reading of *PoM*.

Yet the quasi-Meinongian reading is the only hope for those who would deny the possibility of empty denoting concepts. As far as I am aware, no one in the recent literature has offered a sustained and serious challenge to the rejection of the quasi-

Meinongian interpretation, and that being so our defence of the new orthodoxy will be brief.²⁴

Various arguments have been advanced to challenge the quasi-Meinongian interpretation²⁵. The most obvious is as follows. As Stevens (forthcoming: §1) notes, an ontology does not count as ‘quasi-Meinongian’ simply in virtue of countenancing an ontological realm beyond *existence*. Even after the theory of descriptions was well-established in Russell’s thought—that is, when all sides agree he was *not* quasi-Meinongian—he continued to believe in a realm of non-existents (including universals for instance). Whether or not an ontology is quasi-Meinongian must therefore ‘be a matter of *which kinds* of objects are taken to have being and on what grounds’ (Stevens forthcoming: §1). Objectionable entities include *impossibles*, such as round squares, happily married bachelors and the like. The theory of descriptions obviously enables one to avoid ontological commitment to impossibles, but as the new orthodoxy has repeatedly insisted, the theory of denoting concepts does too.

Russell held that every expression indicates something, but as noted above (§2.2.2), as far as denoting phrases are concerned, this commits him only to the subsistence of the corresponding denoting concepts, not the (putatively) denoted entities. The quasi-Meinongian interpretation illegitimately moves from Russell’s commitment to (say) <the round square> to his commitment to the being of something both round and square. But nothing in *PoM* suggests that he *did* posit a round square (as even Quine acknowledges²⁶). Indeed, in later work, while still committed to the theory of denoting concepts, Russell explicitly indicates an awareness of the resources made available by empty denoting concepts:

‘The present King of England’ is a complex concept denoting an individual; ‘the present King of France’ is a similar complex concept denoting nothing. The phrase intends to point out an individual, but fails to do so: it does not point out an unreal individual, but no individual at all. (*EIP*: 399)

²⁴ Perkins, Jr. (2007) ‘gently disputes’ the thesis. Boukema (2007) also offers a careful analysis of certain aspects of the Russell-Meinong relation.

²⁵ Griffin 1996 contains several. Rebera (2009) suggests that no single argument is individually conclusive, but that together they are compelling.

²⁶ Quine 1966: 5-6.

Endorsing this interpretation of Russell's position requires a slightly careful reading of certain passages of *PoM*, but a slightly careful reading is the least a serious work of philosophy might hope for. For example, Russell says 'Whatever may be an object of thought [. . .] I call a *term*. [E]very term has being, *i.e.* *is* in some sense' (*PoM*: §47, 43). Is this quasi-Meinongian? Only if impossibles genuinely *may* be objects of thought: but why think that? Similarly: 'to deny that such and such a thing is a term must always be false' (*PoM*: §47, 43). Is this quasi-Meinongian? Only if impossibles may be substituted for 'such and such a thing': but why think that?²⁷ Rejecting the quasi-Meinongian interpretation also involves contradicting Russell's memory of his development. He claims, in various places, to have been convinced by Meinongian-style arguments, until he discovered the theory of descriptions.²⁸ Here however, we must simply accept that Russell's memory was unreliable. No aspect of the general position outlined in *PoM* excludes the possibility of empty denoting concepts.

2.2.2.2. *The nature of denoting*

It may be suggested that Salmon and Hylton can maintain their respective positions if we take those positions to acknowledge and accommodate the fact that not all denoting phrases have a denotation. Salmon will thus be understood as attributing to Russell the thesis that definite descriptions are expressions the semantic function of which is to designate the unique entity—if *any*—satisfying their descriptive condition. On this proposal, sentence (1) is about Socrates, but (3) is taken to be a degenerate case, and to be about nothing. Similarly, Hylton will be understood to attribute to Russell a version of (TV Dep) according to which the truth-value of a proposition *p* (containing a denoting concept) is dependent upon the truth-value of the proposition obtained from *p* by replacing the denoting concept by the denoted entity *if there is one*, otherwise *p* is false. On this proposal the otherwise problematic ⟨3⟩ is declared automatically false.

The difficulty, however, is that whatever the independent merits of these positions, they will not do as an interpretation of Russell. For Russell, recall, a denoting phrase enables one to express a proposition which is about a given entity (or entities) in a peculiar way—as *denoted* rather than *referred to* (cf. (Ab1) and (Ab2)). This distinction is what, in *PoM*, secures the distinction between discursive thought and immediate

²⁷ Perkins Jr. (2007: 25) takes a different view of such passages.

²⁸ *MMD*: 13; *MPD*: 84; *Auto*: 455-56.

perception (*PoM*: §56, 53)—between, analogously, general and singular propositions. Now Russell of course accepts that propositions ⟨1⟩ and ⟨2⟩ are intimately linked.

- ⟨1⟩ ⟨<the teacher of Plato> is wise⟩
- ⟨2⟩ ⟨Socrates is wise⟩

They are linked in virtue of the fact that <the teacher of Plato> denotes Socrates. <The teacher of Plato> denote Socrates (rather than anyone else) because (i) Socrates is the unique instance of the predicate (class-concept) *teacher of Plato*, and (ii) <the teacher of Plato> is obtained, *derived* we might say, from the predicate (class-concept) *teacher of Plato*. Associated with every predicate (class-concept) are various denoting concepts obtained from it (as described in Chapter One). Ultimately then, that there is such a proposition as ⟨1⟩ is dependent upon there being such a predicate (class-concept) as *teacher of Plato*, not upon there being some entity that is the unique instance of it (this is no less the case for a proposition such as ⟨3⟩). To be a proposition is to have a truth-value: all and only propositions have truth-values; so if the existence of the proposition ⟨1⟩ is not contingent upon the existence of Socrates (but rather upon the existence of the predicate *teacher of Plato*), then neither, ultimately, is its having a truth-value. This is a round-about way of saying that the proposition is object-independent; for its truth-value is not, ultimately, a matter of how things stand with Socrates, but with the unique instance (if any) of the predicate (class-concept) *teacher of Plato*. And if ⟨1⟩ is object-independent, then ‘the teacher of Plato’ is not a singular term. Generalising, definite descriptions are not singular terms.

Alternatively, we can argue in the following manner. Russell insists that the logical differences pertaining to the different kinds of denoting concepts (<all *F*>, <some *F*>, etc.) are traceable to differences in the kinds of entities denoted, rather than the denoting relation itself (*PoM*: §62, 61–2). Thus all denoting phrases bring it about that the sentences in which they occur express propositions that are about whatever it is that they are about *in the same kind of way*. Consider now a predicate (class-concept) having only one instance, e.g. *author of Waverley*. As Russell says, ‘The word *the*, in the singular, is correctly employed only in relation to a class-concept of which there is only one instance’ (*PoM*: §63, 62). He does not, then, take it that the predicate (class-

concept) from which <the author of *Waverley*> is obtained must be *unique author of Waverley*, but just *author of Waverley*. The denoting concept <the author of *Waverley*> then denotes the unit class, if any, whose member is the entity satisfying the descriptive condition. Now whether this entails that the proposition <<the author of *Waverley*> is Scotch> is about a certain unit class, or about Scott, or about both, is a matter of whether one identifies a unit class with its member—a matter upon which Russell vacillates, eventually coming to distinguish them (*PoM*: §106, 106; cf. §69, 68).²⁹ The crucial point is that the account of how <<the author of *Waverley*> is Scotch> comes to be about whatever it is about, is intended by Russell to be the same as the account of how, say, <<an author of *Principia Mathematica*> smokes> comes to be about whatever *it* is about: i.e. through the relation of denoting, as given in (Ab2). The relevant logical differences between the two propositions pertain to the character of the entities denoted by the respective denoting concepts, not the way in which they are denoted. Now ‘an author of *Principia Mathematica*’ is plainly not a singular term, and since ‘the author of *Waverley*’ functions in so similar a way, *it* is not a singular term either.

Russell does not, then, take definite descriptions as singular terms in *PoM*. But what then of the amended version of (TV Dep) that one might offer Hylton? Once we no longer understand Russell to have taken definite descriptions for singular terms, part of the motivation for (TV Dep) is removed. The principle was proposed as an explanation of the fact that a proposition may be about an entity it does not contain as a constituent (Hylton 1990: 251). In its amended form, and without definite descriptions as singular terms, the principle is now a less than comprehensive explanation, offering no real account of the fact that <3> is not about any of its constituents. It is true that Russell himself offers no real explanation of this; but in extending his position to redress this oversight, the natural move is to make use of his notion of denoting concepts as obtained from predicates (class-concepts). We might then take the denotation of <the present king of France> to be the class of present kings of France, which, taken in extension, is the null class.³⁰ This proposal has two advantages over the amended version of (TV Dep). First, it is far more in keeping with the spirit of Russell’s

²⁹ Any concern that <<the author of *Waverley*> is Scotch> is about a class rather than a man is misplaced. The unit class of authors of *Waverley* is, taken in extension, identical with Scott.

³⁰ This is obviously reminiscent of Frege in the *Grundgesetze* (Frege 1964). Frege *did* take definite descriptions as singular terms. For the relevant differences between the theory of *PoM* and Frege’s account in the *Grundgesetze*, see §2.3 below.

position; second, Russell's truth-primitivism is left uncompromised. (I am not suggesting that Russell implicitly endorsed the modification I have suggested.)

I see, therefore, no reason to suppose that the Russell of *PoM* was committed to either the thesis that definite descriptions are singular terms or (TV Dep). If this account is correct, then the interpretations of the GEA forwarded by Salmon, Hylton, and any other interpreter relying upon these claims, rest on a mischaracterisation of the position against which the Russell of *OD* was arguing.

2.2.3. *Consequences for our understanding of the GEA*

In this section I want to re-emphasise the respects in which Russell's position in *OD* represents an advance from his position in *PoM*, and to indicate how the acknowledgment of these might influence the way in which we approach the task of understanding the GEA.

The great similarity between the theory of denoting concepts and the theory of descriptions is that both are theories of *generality*. As I have argued above, this holds true not only for the relatively uncontroversial quantifier phrases—'all *F*', 'any *F*', 'some *F*', etc.—but also for definite descriptions. In this very limited respect, then, there is no real change. Broadening our view however, *OD* heralds significant advances in both ontological and semantic respects.

The ontological advance is not, as was once thought, that a whole realm of non-existent entities are jettisoned, but, rather, that denoting concepts are abandoned—though one must remember that, as Hylton (1990: 255-56) notes, the variable, which is very arguably a kind of denoting concept, remains in *OD*, though now taken as fundamental (*OD*, 42).³¹ The main action comes, as we have seen in Chapter Three, on the semantic front, where *OD* sees the abandonment of Russell's former naivety concerning the relationship between the surface grammatical form of ordinary language sentences and the logical form of the propositions they express. That a sentence is of subject-predicate form, for example, is no sure indication that the proposition it expresses has the analogous logical form. It is the overcoming of this naivety—the

³¹ This point is discussed again in Chapter Five.

recognition that compositional analysis must be preceded by a stage of SR-interpretive analysis—that constitutes the real advance from *PoM* to *OD*. This development amounts to the recognition of a class of incomplete symbols. In casting definite descriptions as incomplete symbols, one denies that they are singular terms. But as we have seen, it does not follow from the fact that an expression is *not* an incomplete symbol, that it *is* a singular term; nor does it follow from the fact that an expression is *not* a singular term that it *is* an incomplete symbol. We must not lose sight of the fact that the really significant development in *OD* involves the casting of definite descriptions as incomplete expressions, not the denying that they are singular terms.

How might these considerations help us in understanding the GEA? In treating denoting phrases as incomplete symbols, Russell is able to answer the Central Question of *OD* without recourse to denoting concepts (bearing in mind the aforementioned caveat concerning the status of the variable).³² My suggestion therefore, is that the GEA should be understood as criticising the theory of denoting concepts on just this matter: that its answer to the Central Question of *OD* relies upon denoting concepts (denoting complexes, meanings, *Sinne*). On this suggestion then, the tangle that the GEA uncovers concerns the relation between a denoting concept and its denotation.

Now there is nothing very new in this proposal yet. But, the suggestion continues, the kind of problem we should expect to find in the GEA is not so much that denoting concepts are themselves inherently problematic (though they may be³³), but that the theory that posits them is unable to provide a satisfactory explanation of their relation to the entities they denote. The conclusion that any interpretation of the GEA ought to identify is that the theory of denoting concepts (or the theory of meaning and denotation, or Frege's theory of *Sinn* and *Bedeutung*) cannot adequately explain why, say, ⟨the author of *Waverley* is Scotch⟩ is about Scott.

³² The Central Question of *OD* being: How is it that a sentence containing a denoting phrase comes to be about whatever it is about?

³³ I have no wish to deny that there may be such issues, or even that, at certain stages of the GEA, they come under consideration. But, if my suggestion is along the right lines, we should take very seriously the possibility that such issues do not constitute the heart of the matter (for related discussion see Noonan 1996). The central problem identified in the GEA does not concern the denoting concepts themselves, or the possibility of forming propositions directly about them, but rather concerns their relation to their denotations.

2.3. The earlier Russell and Frege compared

In a certain light, Russell's treatment of definite descriptions in *PoM* is superficially similar to Frege's treatment of them in the *Grundgesetze* (Frege 1964).³⁴ It is therefore worthwhile to briefly indicate the differences between the two views whereby Frege, unlike Russell, takes descriptions to be singular terms. This will also serve to emphasise the shared assumption in virtue of which both positions, despite their other differences, are targeted by the GEA.³⁵

Frege introduces the symbol ' \backslash ' as a definite description-forming operator (1964: §11, 49-51). For any concept $\Phi(\xi)$, he takes the expression ' $\backslash \xi \Phi(\xi)$ ' as indicating the *Werthverlauf*—'course of values'—of that concept, and then holds that the expression:

$$(4) \quad \backslash \xi \Phi(\xi)$$

refers to the object falling under the concept $\Phi(\xi)$. Taking $\Phi(\xi)$ to be ' ξ taught Plato', (4) refers to Socrates. For any concept $\Psi(\xi)$ having an extension composed of more or less than exactly one object, the expression:

$$(5) \quad \backslash \xi \Psi(\xi)$$

will have the same reference as ' $\backslash \xi \Psi(\xi)$ ', i.e. the course of values of $\Psi(\xi)$. Taking $\Psi(\xi)$ to be ' ξ is presently king of France', (5) refers to the null-class.

This ensures that all definite descriptions refer to something; and this is necessary because it is highly implausible that a sentence of the form 'the F is G ' fails to be either true or false, as it must do if 'the F ' is a singular term to which nothing answers.³⁶ Given Frege's function-argument framework, if 'the F ' fails to designate an object, there is nothing to stand as argument to the concept given by the predicate ' ξ is G ', and hence no possibility of transition from the concept to a truth-value. In order that all

³⁴ The similarity is particularly obvious if one modifies Russell's position in the manner mooted in §2.2.2.2. I do not, however, think that Russell himself endorsed any such modification.

³⁵ Frege's account of descriptions changes throughout his career. The different positions are outlined in Pelletier & Linsky 2005, and their development plotted by Makin (2000: 173-78).

³⁶ Such, I take it, would have been Frege's position in the *Grundlagen* (Frege 1884: §74n).

sentences of the form ‘the *F* is *G*’ be assigned a truth-value³⁷ every definite description must be assigned a reference. Just as the complex singular term (as Frege conceives it) ‘the father of Charles II’ only refers if ‘Charles II’ does, so, for Frege, ‘the *F* is *G*’ only refers (to a truth-value) if ‘the *F*’ refers.

Russell’s view in *PoM* is significantly different. If there is no *F*, ‘the *F* is *G*’ expresses the false proposition $\langle \text{the } F \rangle \text{ is } G$. It suffices for Russell merely that ‘the *F*’ have a propositional complement (no denotation is technically required): so long as this is so, well-formed sentences containing ‘the *F*’ are assured of expressing a proposition—and hence of being true or false. The propositional complement is the denoting concept $\langle \text{the } F \rangle$, which subsists quite independently of its denoting anything. We *may*, as I suggested above, make the move (Fregean in spirit) of providing a denotation for empty denoting concepts (e.g. the class of *F*s), but it is important to recognise that any such move is *optional* for Russell. Given his truth-primitivism, there is no theoretical objection to the stipulation that any proposition of the form $\langle \text{the } F \rangle \text{ is } G$ is automatically false if $\langle \text{the } F \rangle$ denotes nothing. Intuitively it is not perhaps an attractive move: certainly it appears to be ‘plainly artificial’ (*OD*: 47). But artificiality aside, the *PoM* framework does not demand that every denoting concept denote something.

The difference between the two positions may be emphasised by comparing the levels of analysis to which they appeal (I do not intend ‘analysis’ in any technical sense here). Let us compare the analyses of (6):

- (6) The father of Charles II died on the scaffold.

The Fregean analysis appeals to three levels: the linguistic level, the level of *Sinn*, and the level of *Bedeutung*. For Russell there are only two levels: the linguistic and the propositional.³⁸ At the linguistic level we have the sentence, (6), composed of two

³⁷ As opposed to merely expressing a Thought.

³⁸ Russell’s propositional level is most akin to Frege’s level of *Sinn* as regards judgement: a proposition is the object of judgement for Russell, just as a Thought is for Frege. On the other hand, as regards significance, the propositional level is most akin to the level of *Bedeutung*: propositions are *about* objects (terms, entities).

expressions, a definite description and a predicate. At Russell's second level (the propositional level) we have the proposition expressed, namely $\langle 6_R \rangle$:

$\langle 6_R \rangle$ $\langle \langle \text{the father of Charles II} \rangle \text{ died on the scaffold} \rangle$

For the Russell of *PoM* the analysis does not go beyond this level.³⁹ At Frege's second level (the level of *Sinn*), (6) expresses a Thought, namely $\langle 6_F \rangle$:

$\langle 6_F \rangle$ $\langle \text{\textcircled{\$}the father of Charles II\textcircled{\$}, \text{\textcircled{\$}}\xi \text{ died on the scaffold\textcircled{\$}} \rangle$ ⁴⁰

And for Frege, the truth of $\langle 6_F \rangle$ is a matter of its determining⁴¹ the True at his third level, the level of *Bedeutung*. It will determine the True just in case, if the concept $\Phi(\xi)$ is the determinatum of $\text{\textcircled{\$}}\xi \text{ died on the scaffold\textcircled{\$}}$ and a is the determinatum of $\text{\textcircled{\$}}\text{the father of Charles II\textcircled{\$}}$, the value of the concept $\Phi(\xi)$ for the argument a is the True. This is the root of the difference between Frege and Russell's positions. For Frege, a definite description is employable in the construction of true or false sentences in virtue of having *both* a *Sinn* and a *Bedeutung*: for Russell, a denoting phrase is employable in the construction of true or false sentences solely in virtue of indicating a denoting concept. For Frege—but not for Russell—it is a condition of 'the F is G ' making a truth-evaluatable claim, that 'the F ' designate something. The truth (falsehood) of the sentence 'the F is G ' depends, for Frege, upon the state of the object designated by 'the F ': so for Frege, definite descriptions are singular terms.⁴²

If Frege and the earlier Russell do not both endorse the thesis that definite descriptions are singular terms, what do they have in common? Simply what we identified above: both set about answering the Central Question of *OD* by appeal to the logical relation holding between the propositional complement of a denoting phrase,

³⁹ Obviously I am not claiming that Russell thinks *all* analysis stops here! $\langle 6_R \rangle$ is presumably amenable to further analysis. But in terms of the levels I am discussing here, any further analysis would take place at the propositional level.

⁴⁰ I will use subscripted ' $\text{\textcircled{\$}}$'s, to distinguish *Sinne*, analogously to the use of angle-brackets to distinguish denoting concepts.

⁴¹ I will use 'determining' for the relation between *Sinn* and *Bedeutung*. It is, for the purposes of this thesis, more or less equivalent to *PoM*'s technical sense of 'denoting'.

⁴² Makin (2000: 173) suggests that, in the *Grundgesetze*, Frege was close to recognising that definite descriptions should not be classified as singular terms. If he was, then so much the better for my objections to the Salmon-Hylton position above.

and the denotation of that phrase. As I shall set it out, the GEA is intended to show that the appeal to this machinery cannot provide a satisfactory answer to the central question of *OD*.⁴³

3. The Gray's *Elegy* Argument: An Overview

There are, I think, at least three significant difficulties concerning the GEA. The first is that Russell uses unfamiliar terminology. He speaks, for instance, of *denoting complexes* and speaks sometimes of their *having* a meaning, sometimes of their *being* meanings. Denoting complexes are not mentioned in *PoM*, and the terminology is certainly not Fregean. So what exactly are they? The second difficulty concerns the fact that the argument addresses an issue that Russell had not addressed before (except in *OF*, of course). The third difficulty concerns the fact that while the argument Russell brings to bear is recoverable from the text, it isn't *easily* recoverable. As a result of the second and third difficulties, an interpretation that presents a coherent account of the argument *and* links it to the text will inevitably be somewhat convoluted. In an effort to overcome (or at least mitigate) these difficulties, I propose in this section to give an overview of the GEA. In §4 I will go through the argument in greater detail, following the course set by the text.

The Central Question of *OD* is: How is it that a sentence containing a denoting phrase comes to be about whatever it is about? The GEA concerns the adequacy of the answer that the theory of meaning and denotation is able to provide. Let us begin, then, with that answer. It will help to have an example in mind. So let us ask what explanation the theory of meaning and denotation is able to give of the fact that the sentence 'the teacher of Plato is wise' is about Socrates.

In the most general terms, the explanation is that 'the teacher of Plato is wise' is about Socrates because the propositional complement of its subject term, the definite description 'the teacher of Plato', is an entity with the property of denoting Socrates. A terminological issue may be resolved here: a 'denoting complex' is simply the propositional complement of a denoting phrase. It is more or less equivalent to 'denoting concept', as used in *PoM* (one difference being that while it seems quite

⁴³ Though as I suggest in §5, Frege's position can—in a certain sense—be rescued.

acceptable to speak of a simple denoting concept, it seems odd to speak of a simple denoting complex). Thus ‘the teacher of Plato is wise’ is about Socrates because it expresses a proposition containing a denoting complex—namely <the teacher of Plato>—that denotes him.

In *PoM* there is really no more explanation of the fact that ‘the teacher of Plato is wise’ is about Socrates than we have just given. Russell does, however, hint that there is more to be said. He writes:

In a full discussion, it would be necessary also to discuss the denoting concepts: *the actual meanings of these concepts*, as opposed to the nature of the objects they denote, have not been discussed above. (*PoM*: §65, 65, emphasis added)

The implication here is that denoting concepts (= denoting complexes) *have* meanings. This is surprising since we have been led to believe that a denoting complex *is* the meaning of a denoting phrase. The theory of meaning and denotation is, we supposed, a theory premised upon the distinction between the meaning of a denoting phrase—i.e. a certain denoting complex—and its denotation. That is, the theory distinguishes between what a sentence containing a denoting phrase is *about* and what *occurs in* the proposition expressed by that sentence—i.e. the meaning of the denoting phrase.

Russell’s speaking of the meaning of the propositional complement of a denoting phrase reflects the recognition that there must be more to the theory of meaning and denotation’s response to the Central Question. The reason why there must be more concerns the status of the theory of meaning and denotation as a theory of denoting, i.e. as an account of the possibility of propositions of generality. ‘The teacher of Plato is wise’ is about Socrates because the denoting complex <the teacher of Plato> denotes him; but in addition to this, the theory of meaning and denotation must provide an explanation of why it is that <the teacher of Plato> denotes Socrates rather than anyone else. It isn’t just by accident—it isn’t, for example, just by luck that (<all men> are mortal) is about all men rather than all pencils. Rather, when a denoting complex denotes an entity, it does so because that entity has a certain property, namely that property which we might characterise as the *descriptive content* of the corresponding denoting phrase. The complex <the teacher of Plato> denotes Socrates because it

denotes *whatever* taught Plato and, as a matter of fact, Socrates taught Plato. If Xanthippe had taught Plato, <the teacher of Plato> would denote her; if Plato had never received instruction, <the teacher of Plato> would denote nobody.

To accommodate this aspect of the theory of meaning and denotation, Russell comes to the view that ‘the teacher of Plato is wise’ is about Socrates because <the teacher of Plato> denotes him, and because the meaning of the complex is the descriptive condition *taught Plato*. (We may say—and Russell is disposed to put things this way—that the denoting phrase ‘the teacher of Plato’ *means* <the teacher of Plato>. But since we will also need to speak of the *meaning* of <the teacher of Plato>, it is better to say that the phrase *indicates* <the teacher of Plato>, and to reserve ‘means’ and ‘meaning’ for the *meaning* of complexes.) So in order to provide an adequate answer to the Central Question, we need to take account of the relevance of meaning, as well as denotation. In explaining why ‘the teacher of Plato is wise’ is about Socrates, we need to take account of the meaning of the complex <the teacher of Plato> in order to adequately reflect the fact that the sentence is about Socrates *in virtue of* his having taught Plato.

Russell sometimes speaks of denoting complexes as having ‘two sides’ and being ‘two-fold’ (*OF*: 383). What he means is that denoting complexes are capable of occurring in propositions in more than one way. In general a proposition containing a denoting complex will be about the denotation of that complex (as in <the teacher of Plato> is wise). But sometimes a proposition containing that same complex will be about the complex itself. This latter possibility reflects a fundamental aspect of Russell’s general metaphysical framework: if there were no propositions about complexes in virtue of containing them—if denoting complexes were not possible logical subjects—the ‘contradiction always to be feared’ (*PoM*: §74, 76) would raise its ugly head. Propositions about a given denoting complex and about its denotation will, then, have a common constituent⁴⁴, namely the denoting complex itself. (Russell is lax, it seems to me, in that he sometimes fails to keep the complex suitably distinct from its meaning; I take his position to be that if one wishes to speak about the meaning of a complex, one formulates a proposition about the complex. If the complex and its

⁴⁴ At least when the propositions about the denotation are about it in virtue of containing the complex.

meaning are distinct, then this is either a gross mistake, or requires further explanation. Either way, it seems to me that Russell goes astray in this respect.)

The version of the theory of meaning and denotation under discussion at present is a ‘four-entity’ version of the theory. Henceforth I shall refer to it as ‘the four-entity theory’, or ‘4E theory’ for short. It is a four-entity theory because it posits four distinct entities: (i) the denoting phrase; (ii) the denoting complex; (iii) the meaning; and (iv) the denotation. The 4E theory and the notation I shall use in association with it is given in Fig. 3.

Fig. 3

The Four-Entity (4E) Theory

i. Denoting Phrase.....	‘the F ’
ii. Denoting Complex.....	$\langle \text{the } F \rangle$
iii. Meaning.....	$M\langle \text{the } F \rangle_M$ [= M]
iv. Denotation.....	$D\langle \text{the } F \rangle_D$ [= D]

(I will use ‘ M ’ and ‘ D ’ as singular terms for the meaning and denotation (respectively) of the denoting phrase ‘the F ’. The possibility of such a use of ‘ M ’ will be called into question by the GEA, but I employ it independently of those concerns, solely as a useful heuristic device.)

As mentioned above, on the 4E theory some propositions containing a denoting complex will be about its denotation, but some will be about the complex itself (or about its meaning, Russell not adequately distinguishing these two possibilities). This is reflected in the notation above by the use of subscripted ‘ M ’s and ‘ D ’s. In the notation, we will say that $\langle \text{the teacher of Plato} \rangle$ is the propositional complement of ‘the teacher of Plato’, but will hold that

$$\langle 7 \rangle \quad \langle \langle \text{the teacher of Plato} \rangle \text{ is wise} \rangle$$

is strictly ambiguous. It might be:

⟨7a⟩ ⟨_D<the teacher of Plato>_D is wise⟩

in which case it is a proposition about Socrates; or it might be:

⟨7b⟩ ⟨_M<the teacher of Plato>_M is wise⟩

in which case it is a proposition about <the teacher of Plato>. The crucial point is that although ⟨7a⟩ and ⟨7b⟩ are not equivalent, they have a common constituent in entity-position (i.e. <the teacher of Plato>).

The notation we have introduced provides a convenient way of illustrating the claim that denoting complexes are ‘twofold’. This is welcome because the theory of meaning and denotation’s response to the Central Question relies upon the claim that denoting complexes are twofold; for it relies upon the claim that the complex occurring in entity-position in ⟨7a⟩ denotes Socrates in virtue of the fact that its meaning embodies a descriptive condition that Socrates satisfies.

However, in appealing to the relevance of *meaning* in this way, one in fact merely provides the beginnings of an adequate response to the Central Question. To provide a full answer, we would need to say a little more about the 4E theory. How exactly is it that the meaning of <the teacher of Plato> comes to be relevant in ⟨7a⟩? What is the nature of the connection between the meaning of a complex and that complex itself, i.e. between _M<the teacher of Plato>_M and <the teacher of Plato>? In virtue of what is it possible for a complex to occur in such a way that a proposition is about its denotation *and* also occur in such a way that a proposition is about itself or its meaning? Our notation provides convenient ways of representing these relations, but what is needed is an account of what it is that the notation represents. The validity of the notation is, so to speak, *underwritten* by the nature of the relations that hold at the logical level (the propositional level); hence we are only entitled to introduce the notation at all if we have a firm grasp of the relations it represents.

Russell will argue in paragraph (D) of the GEA that there is, in fact, no non-circular way of explaining what the notational operations that we perform represent. Of course the use of subscripted ‘M’s and ‘D’s is my proposal. Russell employs, to the same

effect, inverted commas. The use of inverted commas is probably the most notorious aspect of the GEA. Its notoriety is certainly merited, but we can, I think, recover the points that Russell wishes to make.

The upshot of paragraph (D) will be that the 4E theory fails to adequately explain what the use of subscripts (or inverted commas) represents, and hence that it is unable to justify its claim that denoting complexes are twofold. In the light of this, Russell proposes a different version of the theory of meaning and denotation. This second version may be thought of as a ‘three-entity theory’ (‘3E theory’). On this theory—which is far closer to Frege’s position than the 4E theory was—the denoting complex and meaning are taken to be *identical*, leaving us only three entities: (i) the denoting phrase; (ii) the meaning (= denoting complex); and (iii) the denotation.

Fig. 4

The Three-Entity (3E) Theory

i. Denoting Phrase.....	‘the <i>F</i> ’	
ii. Meaning (=Denoting Complex).....	$M\langle \text{the } F \rangle_M$	[= \mathbb{M}]
iii. Denotation.....	$D\langle \text{the } F \rangle_D$	[= \mathbb{D}]

The transition from the 4E to the 3E theory occurs in paragraphs (E) and (F). At the end of paragraph (F) and the beginning of paragraph (G), Russell indicates an initial problem facing the 3E theory. If one wishes to formulate a proposition about a meaning, one must denote it with a second meaning; but identifying this second meaning turns out to be problematic.

This initial problem is not the main objection to the 3E theory, however. The major objection—signalled in the second half of (G) and developed in (H)—brings us back to the Central Question. Russell now argues that if meaning is not an aspect of a denoting complex (as, by intention at least, it was on the 4E theory) there is no explaining how meaning can be ‘relevant’ to the fact that ‘the teacher of Plato is wise’ is about Socrates. If denoting complexes *merely* denote (and do not also *mean*) then although the proposition $\langle \text{the teacher of Plato} \rangle$ is wise is about Socrates, there is no accounting for the fact that it is about him only in virtue of his having taught Plato. But this is a fundamental requirement of the theory of meaning and denotation: if it cannot satisfy it,

it must give up any pretensions to being an adequate account of propositions of generality. Its response to the Central Question will be entirely inadequate, and the theory must, on this ground, be abandoned.

4. The Gray's *Elegy* Argument: An Interpretation

Although I have not attempted to provide detailed discussion of rival interpretations of the GEA, I have on occasion indicated where my interpretation differs significantly from others. These comments are (with regret) relegated to footnotes, for to include them in the main text would clog up and complicate a discussion that is already complicated enough. The main focus in these footnotes are the interpretations of Makin (2000) and Levine (2004), for it is these interpretations which—as far as I can tell—have influenced mine the most.⁴⁵

4.1. Paragraphs (A) and (B): introduction

The relation of the meaning to the denotation involves certain rather curious difficulties, which seem in themselves sufficient to prove that the theory which leads to such difficulties must be wrong. (OD: 48 A)

The opening paragraph is perfectly clear. It is the *relation* of meaning to denotation that will come under investigation. We should not therefore, expect the GEA to call into question either the ontological status of meanings, the possibility of formulating propositions about them, or the possibility of epistemic access to them.⁴⁶

The difficulty begins in (B), which is as confused as (A) is clear.

When we wish to speak about the *meaning* of a denoting phrase, as opposed to its *denotation*, the natural mode of doing so is by inverted commas. Thus we say:

The centre of mass of the solar system is a point, not a denoting complex;
'The centre of mass of the solar system' is a denoting complex, not a point.

⁴⁵ I would like to record my debt to *many* excellent discussions of the GEA, notably: Geach 1959; Cassin 1970; Dau 1985; Cartwright 1987b; Hylton 1990; Pakaluk 1993; Wahl 1993; Kremer 1994; Noonan 1996; Landini 1998b; Makin 2000; Levine 2004; and Salmon 2005.

⁴⁶ Contrast, e.g., Levine 2004: 265-66. For interpretations with an epistemological twist see: Kremer 1994 or Noonan 1996.

Or again,

The first line of Gray's Elegy states a proposition.
 'The first line of Gray's Elegy' does not state a proposition.

Thus taking any denoting phrase, say *C*, we wish to consider the relation between *C* and '*C*', where the difference of the two is of the kind exemplified in the above two instances. (*OD*: 48-9 B)

Alonzo Church famously claims that the objection raised in the GEA is 'traceable merely to confusion between use and mention of expressions, of a sort which Frege is careful to avoid by the employment of quotation-marks' (Church 1943: 302). Russell's use of inverted commas is certainly confusing, but Church is simply wrong to think that the objections 'completely vanish' (Church 1943: 302) when they are applied consistently and correctly. In (B) at least, the main difficulty does not so much concern the use of inverted commas, but an equivocation between the 4E and 3E theories.

Russell's first example (shortened slightly), may be rendered as follows:

- (8) The centre of mass of the solar system is a point.
 ⟨8⟩ ⟨_D<the centre of mass of the solar system>_D is a point⟩

If *p* is the point which is centre of mass of the solar system, ⟨8⟩ expresses a proposition about *p* in virtue of its being denoted by <the centre of mass of the solar system>. (9), on the other hand, ought to express a proposition which is not about *p* but rather the meaning of the phrase 'the centre of mass of the solar system'⁴⁷.

- (9) 'The centre of mass of the solar system' is a denoting complex.

The difficulty is that if we interpret (9) in accordance with the 4E theory, yielding ⟨9⟩,

- ⟨9⟩ ⟨_M<the centre of mass of the solar system>_M is a denoting complex⟩

⁴⁷ These inverted commas are 'phrase-mentioners': the inverted commas in (9) are Russell's 'meaning-indicators'.

the proposition is, as was intended, about the meaning in question, but—as was not intended—the proposition is false. For ${}_M\langle\text{the centre of mass of the solar system}\rangle_M$ is a meaning, not a denoting complex (meanings and denoting complexes not being identified on the 4E theory).

One option here—which challenges my entire approach—is to suggest that the imposition of the 4E theory was in the first place unwarranted, to suggest that it had no place in either *PoM* or Frege’s theory, and to claim that it has been posited merely in order to make sense of an otherwise intractable argument. The less interesting, but more accurate, reading, it will be said, is that the GEA is simply confused and incoherent.⁴⁸

However this objection to my general approach can and should be resisted. Three points are salient. Firstly, as noted in §3, *PoM* contains at least a hint that Russell had something like the 4E theory in mind, as Russell there distinguishes between a denoting concept and the meaning of that concept (*PoM*: §65, 65). Secondly, the fact that the 4E theory is not to be found in Frege is neither here nor there. The GEA will address both 4E and 3E theories (Frege’s is a 3E theory). Thirdly, the 4E theory is explicit in *OF*, where we find the origins of the GEA. There are far too many examples to quote them all. Here are a few:

When *a concept has meaning and denotation*, if we wish to.... (*OF*: 381, emphasis added)

The endeavour to speak about the meanings of denoting complexes leads.... (*OF*: 382)

...the necessity of distinguishing *the two sides in complexes*. (*OF*: 383, emphasis added)

One might try to explain away such evidence by claiming that Russell in fact intends ‘denoting complex’ as synonymous with ‘denoting phrase’. But this is highly unlikely. In the first of the above quotations he is distinguishing the *concept*—not complex—from the meaning, and he certainly did not intend ‘denoting concept’ as synonymous with ‘denoting phrase’ (and a similar claim is made, as we have seen, at

⁴⁸ Something like this general approach is found, in spirit at least, in Urquhart 2005.

the end of the discussion of denoting in *PoM*). Moreover, the wording of (B) strongly suggests that ‘denoting phrase’ and ‘denoting complex’ are to be distinguished. He writes: ‘When we wish to speak about the *meaning* of a denoting phrase’, and then gives an example in which what is spoken of is a denoting complex—suggesting that phrase and complex are not to be conflated. Thus I take it that the 4E theory is obviously present in *OF*, implicit in *PoM*, and at stake in the GEA.

The correct explanation of paragraph (B) is, I suggest, that Russell has simply presented the position poorly. (9) is best understood in terms of the 3E theory, according to which (9) is true. This reading is supported by *OF*, in which Russell introduces the same examples, commenting that:

In each of these [i.e. (8) and (9)] the subject occurs as entity, not as meaning; in [(9)], the subject is “*C*” [i.e. $M<\text{the centre of mass of the solar system}>_M$], in [(8)] it is *C* [i.e. $D<\text{the centre of mass of the solar system}>_D$]. Thus it would seem that “*C*” and *C* are *two different entities*. In that case, what is the connection between them? (*OF*: 383, emphasis added)

Here Russell uses the same examples as in (B) to illustrate and motivate the 3E theory, stressing that ‘*C*’ and *C*—that is: $M<\text{the centre of mass of the solar system}>_M$ and $D<\text{the centre of mass of the solar system}>_D$ —are *different entities*, rather than the same entity under different modes of occurrence.

At the end of (B) Russell attempts to frame his distinctions in general terms, using the third letter of the alphabet capitalised as a schematic letter abbreviating any denoting phrase. (Henceforth, when I wish to mention one of Russell’s deployments of the schematic letter ‘*C*’ I will either use the phrase ‘schematic “*C*”’ (or some clear variant), or indicate (in parentheses or a footnote) whether I am using inverted commas as normal ‘phrase-mentioners’ or as Russellian ‘meaning-indicators’.) He begins badly, mentioning a phrase without using any device at all (he can’t use inverted commas, having just proposed to use them as ‘meaning-indicators’):

Thus taking any denoting phrase, say *C*... (*OD*: 49 B)

and continues:

...we wish to consider the relation between C and ' C '. (OD: 49 B)

The second schematic ' C ' in this last quotation employs inverted commas according to Russell's stated convention (as meaning-indicators). The first schematic ' C ', given plain (i.e. without inverted commas), could indicate the denoting complex itself (i.e. the denoting complex posited on the 4E theory), or the denotation (i.e. \mathbb{D}). On the former interpretation, Russell wishes to consider the relation between <the centre of mass of the solar system> and M <the centre of mass of the solar system> M , which would be in accord with an investigation into the 4E theory. On the latter interpretation, Russell wishes to consider the relation between p and M <the centre of mass of the solar system> M . Both interpretations are independently plausible and so there is no great pressure to decide between them. Only one reading should be guarded against: the plain instance of the schematic ' C ' in the most recently quoted passage is not intended to mention the denoting phrase (as is the first plain instance of schematic ' C ' in (B)). If it were, we could make no sense of the final claim, that 'the difference of the two is of the kind exemplified in the above two instances'.

4.2. Paragraph (C): statement of the argument

In paragraph (C) the overall structure of the GEA is presented, albeit not very clearly.

We say, to begin with, that when C occurs it is the *denotation* that we are speaking about; but when ' C ' occurs, it is the *meaning*. Now the relation of meaning and denotation is not merely linguistic through the phrase: there must be a logical relation involved, which we express by saying that the meaning denotes the denotation. But the difficulty which confronts us is that we cannot succeed in *both* preserving the connexion of meaning and denotation *and* preventing them from being one and the same; also that the meaning cannot be got at except by means of denoting phrases. This happens as follows. (OD: 49 C)

Russell begins by comparing two occurrences of a denoting complex in a proposition. In both cases, as urged in OF⁴⁹, the complex occurs in an entity-position in the proposition. In one case, <8>, the proposition is about the denotation of the complex, in the other case, <9>, the proposition is about the meaning of the complex.

⁴⁹ OF: 383.

- (8) $\langle_D \langle \text{the centre of mass of the solar system} \rangle_D \rangle$ is a point
- (9) $\langle_M \langle \text{the centre of mass of the solar system} \rangle_M \rangle$ is a denoting complex

Russell now says that the relation between $\langle_M \langle \text{the centre of mass of the solar system} \rangle_M \rangle$ and $\langle_D \langle \text{the centre of mass of the solar system} \rangle_D \rangle$ —is not ‘merely linguistic through the phrase’. The point here is almost universally held to be that although it is possible to draw a relation between meaning and denotation in terms of their respective relations to a common denoting phrase, this ‘merely linguistic’ relation is less fundamental than, and derivative upon, the *logical* relation in which they stand to one another. That is, although the logical relation of denoting can be given as the relative product of the phrase-meaning and phrase-denotation relations, this relation is indirect, when what is required is a characterisation of the *direct* relation of meaning to denotation.⁵⁰ I suspect, however, that a different reading may be more apt.⁵¹

According to my alternative reading, Russell’s ‘linguistic through the phrase’ point concerns the relation between the expressions “‘C’” and ‘C’—i.e. schematic ‘C’ in inverted commas and schematic ‘C’ plain; or in our notation, between the expressions ‘ $\langle_M \langle \text{the centre of mass of the solar system} \rangle_M \rangle$ ’ and ‘ $\langle_D \langle \text{the centre of mass of the solar system} \rangle_D \rangle$ ’. His point is that the notational convention that we introduce (be it subscripted ‘M’ and ‘D’, or inverted commas) is a representation of something more fundamental at the logical level. We use subscripted ‘M’ to indicate that the complex occurs in such a way that the proposition is about the meaning of the denoting phrase, or use subscripted ‘D’ to indicate that the complex occurs in such a way that the proposition is about the denotation of the denoting phrase. That is fine as far as it goes: we are quite at liberty to introduce any such notational convention we like. But a condition of doing so is that we genuinely understand what it is that our notation represents. Thus Russell’s point might be put like this: that we indicate the relation between the meaning and denotation of a denoting complex by using different

⁵⁰ This is clearly indicated in diagrammatic form by Demopoulous (1999). Cf. Figure 2 in Chapter Two (§4.3).

⁵¹ As far as I know, no other commentator has offered the following reading of the ‘linguistic through the phrase’ claim.

subscripts attached to the expression for the complex is justified only to the extent that we are familiar with the logical operations that the linguistic operations track.⁵²

The remainder of paragraph (C) (less the final clause, to be discussed below) sets out the crux of the argument. The difficulty, apparently, is that we cannot realise—as presumably we must—both of the following two aims:

- To preserve the connection of meaning and denotation;
- To prevent the meaning and denotation from being one and the same.

The difficulty, however, is to discern any part of the GEA in which it seems as if there is a danger of the meaning and denotation becoming one and the same. It is clear that such an event would be disastrous for any version of the theory of meaning and denotation (since the theory is premised upon their distinction); but there is little indication from any of the eight paragraphs of the argument that this might be on the cards.⁵³ What then could Russell have in mind? I suggest that the incompatible aims are in fact the following:

- To preserve the connection of meaning and denotation;
- To prevent the meaning and denotation from being ‘two sides’ of the one complex (alternatively: to prevent the denoting complex from having the ‘two sides’ of meaning and denotation).

This re-description of the second aim may seem a little far removed from the actual text of paragraph (C). But that this kind of thought is in the offing is suggested in *OF*, where Russell adverts:

⁵² This interpretation of the ‘linguistic through the phrase’ claim enables a deeper understanding of paragraph (D), I will suggest below.

⁵³ Salmon describes the central thrust of the argument as follows:

Here is the chestnut in a nutshell: The seemingly innocuous thesis that definite descriptions are singular terms is untenable. For the attempt to form a proposition directly about the content of a definite description (as by using an appropriate form of quotation) inevitably results in a proposition about the thing designated instead of the content expressed. I call this phenomenon the *Collapse*. (Salmon: 2005: 1071)

One might take the Collapse to be *an instance of meaning and denotation becoming one and the same*; but the italicised phrase doesn’t strike me as a natural way of describing it. (I also dispute Salmon’s claim that there *is* a Collapse.)

the indissolubility of meaning and denotation, and the impossibility of inventing a symbolism which will avoid the necessity of distinguishing the two sides in complexes. (*OF*: 383)

On this reading, the second aim now amounts this: that we cannot prevent denoting complexes from having the two sides. But one might think: ‘Well what is so wrong with that: isn’t that *exactly* what the 4E theory claims?’ And indeed that *is* exactly what the 4E theory claims; but Russell’s contention will be that:

(GEA 1) If the meaning and denotation are two sides of the one denoting complex, then we cannot explain (‘preserve’) the connection between them.

If Russell could establish (GEA 1), we would have to abandon the 4E theory; and indeed this is what Russell does in paragraph (E), where he switches from the 4E theory to the 3E theory, on the basis of the argument presented in (D).

However, in (F), (G) and—especially—in (H), Russell is going to argue that the 3E theory is even worse off. If the meaning and denotation are completely distinct entities, there is even less chance of satisfactorily explaining their relation (or so he will argue). From this we may understand him as implicitly drawing the moral that only way to preserve a satisfactory connection between meaning and denotation is to reinstate the 4E view, since:

(GEA 2) If the meaning and denotation are *not* two sides of the one denoting complex, then we cannot explain (‘preserve’) the connection between them.

Together (GEA 1) and (GEA 2) form the premises of an argument whose conclusion is:

(GEA 3) We cannot explain (‘preserve’) the connection of meaning and denotation.

The Central Question of *OD* is ‘how is it that sentences containing denoting phrases come to be about whatever it is that they are about?’ The theory of meaning and denotation offers an answer based on the connection between meaning and denotation. Thus if (GEA 3) could be demonstrated, the theory of meaning and denotation would have no response to the Central Question of *OD* and would therefore have to be abandoned. The demonstration of (GEA 3) is, as I read it, the aim of the GEA.⁵⁴

The final clause of paragraph (C), ‘that the meaning cannot be got at except by means of denoting phrases’ (*OD*: 49 C) does not, I think, introduce any new dimension to the argument. The fact that, on the 3E theory, ‘the meaning cannot be got at except by means of denoting phrases’ will be shown to be problematic in (H). But there it will be taken as symptomatic of a deeper problem. That Russell tacks the clause onto the end of (C) suggests that he has a different objection in mind. Most likely, the objection is that, if meanings can only be ‘got at [. . .] by means of denoting phrases’, then they cannot be logical subjects (since no proposition could be about a meaning in virtue of containing it). This is almost universally held to be the (or at least *a*) central issue in the GEA. I would suggest, however, that this issue is subordinate to the more fundamental issue, which is the problem regarding the relation of meaning and denotation.⁵⁵

4.3. Paragraph (D): against the 4E theory

We come now to the argument itself.

The one phrase *C* was to have both meaning and denotation. But if we speak of ‘the meaning of *C*’, that gives us the meaning (if any) of the denotation. ‘The meaning of the first line of Gray’s *Elegy*’ is the same as ‘The meaning of “The curfew tolls the knell of parting day”’, and is not the same as ‘The meaning of “the first line of Gray’s *Elegy*”’. Thus in order to get the meaning we want, we must speak not of ‘the meaning of *C*’, but of ‘the meaning of “*C*”’, which is the same as ‘*C*’ by itself. Similarly ‘the denotation of *C*’ does not mean the denotation we want, but means something which, if it denotes at all, denotes

⁵⁴ Pakaluk 1993 also recognises that the argument begins with one view, identifies a problem with it, and so moves on to another view which is also rejected. Our interpretations differ greatly in other respects.

⁵⁵ Two considerations ought to give anyone who thinks that the problem of meanings as logical subjects is central to the GEA pause. Firstly, if this is the central issue, why does Russell say in (A) that the central problem concerns the *relation* of meaning and denotation? Secondly, given that the problem of meanings as logical subjects is just an instance of ‘the contradiction always to be feared’ (*PoM*: §74, 76), it is extremely surprising that Russell was unable to make his point more clearly than he managed in the GEA: he was, by 1905, well-practised in this kind of worry.

what is denoted by the denotation we want. For example, let ‘*C*’ be ‘the denoting complex occurring in the second of the above instances’. Then

C = ‘the first line of Gray’s Elegy’, and

the denotation of *C* = The curfew tolls the knell of parting day. But what we *meant* to have as the denotation was ‘the first line of Gray’s Elegy’. Thus we have failed to get what we wanted. (*OD*: 49 D)

In explaining how it is that, say, ‘the centre of mass of the solar system is a point’ comes to be about *p*, the 4E theory appeals to the following facts. Firstly, ‘The centre of mass of the solar system’ has for its propositional complement the denoting complex <the centre of mass of the solar system>. Secondly, this complex denotes *p* in virtue of *p*’s uniquely satisfying the property embodied by the meaning of the complex, namely *being a centre of mass of the solar system*. Since the 4E theory invokes the twofold nature of denoting complexes, it owes an explanation of it. The most obvious manifestation of the twofold nature of denoting complexes is that some propositions containing a denoting complex are about its denotation, while some are about its meaning⁵⁶. The 4E theory certainly gives the impression of being able to account for this. A simple notational device (inverted commas, or subscripts) is used to alert us when a complex occurs in a proposition in such a way that the proposition is about the meaning of the complex, rather than the denotation. But if this is to be anything more than a mere impression, we require an explanation of precisely what it is that the notational device represents. As I understand it, paragraph (D) is intended to demonstrate that the 4E theory’s explanation involves a circularity.

We begin by interpreting Russell’s examples in terms of the 4E theory. As a first stab the most plausible interpretation is as follows (I will not ultimately endorse this interpretation). The first sentence of (D) indicates that Russell is discussing denoting phrases (as opposed to complexes), in particular the denoting phrase which is represented by the schematic ‘*C*’. When the expression ‘the meaning of *C*’—that is, the result of attaching the prefix ‘the meaning of ...’ to the schematic ‘*C*’—occurs as the grammatical subject of a sentence, that sentence does not express a proposition about the meaning of the expression represented by the schematic ‘*C*’, but of the denotation of

⁵⁶ Or about the complex itself—Russell does not seem to keep these as clearly separate as he ought to.

that expression. On this view, (10) and (11) express equivalent propositions (i.e. propositions making the same assertion of the same entity):

- (10) The meaning of the first line of Gray's *Elegy* is thus-and-so.
- (11) The meaning of 'the curfew tolls the knell of parting day' is thus-and-so.⁵⁷

But neither of these, Russell maintains, expresses a proposition about the meaning of the denoting phrase 'the first line of Gray's *Elegy*'. In order to formulate such a proposition, the denoting phrase must be prefixed with 'the meaning of ...' and *mentioned*, not used.

Now if one had understood the 'linguistic through the phrase' claim in paragraph (C) to mean that the relation between meaning and denotation cannot be explained in terms of their respective relations to their common denoting phrase, the above interpretation of the first half of (D) would be extremely plausible. The problem identified will be that the only way to speak about a meaning is to mention the denoting phrase that expresses it; and that plainly invokes a linguistic relation, rather than a logical one. But as suggested above (§4.2), I do not think that this is the best interpretation of the 'linguistic through the phrase' claim. That claim should rather be understood as the demand for an account of what, at the logical level, is represented, at the linguistic level, by the enclosing of an expression for a denoting complex in inverted commas. Now this may not, in general terms, appear to differ significantly from the interpretation I have rejected. Are they not just two ways of expressing the same point?

They are not. On the 4E theory the relation of meaning and denotation is to be explained in terms of their relation to a common denoting *complex* (as opposed to *phrase*). It is the different ways in which this complex can occur in a proposition that requires investigation; and it is this difference of mode of occurrence that is represented by Russell's inverted comma (or our subscript) notation. Hence Russell's point in (D) concerns the modes of occurrence of denoting complexes in propositions, rather than the use and mention of denoting phrases in sentences. These are, to a certain extent,

⁵⁷ The inverted commas here are used to mention the line of poetry.

analogous; but they are not identical. In particular, Russell is not arguing that there is anything inherently problematic about expressing a proposition about the meaning of a denoting phrase by mentioning that phrase. This practice is perfectly acceptable as far as it goes. But it does not constitute an explanation of what it represents—and why would it? It aims only at representing, at labelling, something: not at explaining it. It is the *explanation* that Russell is probing in the GEA, and in (D) in particular. That explanation concerns the different modes of occurrence of a complex in a proposition. So we should expect (D) to concern these matters, not purely linguistic matters concerning use and mention.⁵⁸

In our notation we use subscripted ‘D’ or ‘M’ to indicate the mode of occurrence of a complex in a proposition. Enclosing a denoting complex in subscript ‘M’ allows us to speak about the meaning of that complex: for example,

- $\langle_M \text{the } F \rangle_M = \mathbb{M}$

If this is correct, then Russell’s inverted comma notation—our subscript ‘M’ notation—represents, at the propositional level, a function f from complexes to their meanings.

The problem Russell identifies in the first half of (D) stems from the fact that the argument-places of functions are entity-positions. That is to say, our function f has the following structure:

- $_m[f]_{m/e}[x]_e$

Presenting the denoting complex $\langle \text{the } F \rangle$ to this function, we get:

- $_m[f]_{m/e}[\langle \text{the } F \rangle]_e$

⁵⁸ Like Levine (2004), I contend that the third word of (D) should be ‘complex’ rather than ‘phrase’. Levine implies that Russell’s use of ‘phrase’ was intentional, and that it meant ‘complex’ (in accordance with the general Russellian tendency to use words for linguistic items for their non-linguistic correlates). He takes Russell’s use of ‘denoting phrase’ in the *OD* passages after the GEA to indicate a contrast with its earlier employment in the GEA (Levine 2004: 269-70). This strikes me as extremely unlikely; I prefer to attribute to Russell a mistake here.

But on the 4E theory this formula is ambiguous, since ‘<the F >’ could be either of ‘ $\langle_D \text{the } F \rangle_D$ ’ or ‘ $\langle_M \text{the } F \rangle_M$ ’. Denoting phrases are normally used to express propositions about their denotations. This gives us:

- ${}_m[f]_{m/e}[\langle_D \text{the } F \rangle_D]_e$

which is equivalent to:

- ${}_m[f]_{m/e}[\mathbb{D}]_e$

But we seek an explanation of how it is that the subscript notation allows us to formulate propositions about meanings; and the above formula will not enable us to speak about the meaning we want (i.e. \mathbb{M}), but only about the meaning of \mathbb{D} (if \mathbb{D} even has one). This is what Russell means by saying: ‘Thus in order to get the meaning we want, we must speak not of ‘the meaning of C ’’, and is what is shown by the equivalence of the propositions expressed by (10) and (11).

One way out of this problem is to fill the entity position in ${}_m[f]_{m/e}[x]_e$ with a denoting complex that denotes the meaning we want. But on the 4E theory, we don’t want to introduce a wholly separate entity, but rather the original denoting complex, under a different mode of occurrence. This, then, is what the subscript ‘M’ and ‘D’ notation facilitates. A denoting complex has two possible ways of occurring as entity: *denotatively* (i.e. enclosed in subscript ‘D’s) or *non-denotatively* (i.e. enclosed in subscript ‘M’s). When a complex occurs denotatively in an entity-position in a proposition, the proposition will be about the denotation of the complex; when a complex occurs there non-denotatively, the proposition will be about the meaning of the complex.⁵⁹

⁵⁹ At this stage I should point out a difference between my interpretation of (D) and another that may otherwise appear similar. I have distinguished between denotative and non-denotative occurrences of complexes. A similar distinction is made by Levine (2004: 272-73). However, in making the distinction I allow that $\langle_M \text{the } F \rangle_M$ is thus-and-so and $\langle_D \text{the } F \rangle_D$ is thus-and-so contain the same entity in entity-position and yet are about different entities, and hold this to be a key commitment of the 4E theory. Levine, on the other hand, attributes to Russell the following view as an assumption:

AS* Whenever [a denoting complex] occupies a subject-position [i.e. entity-position] in a propositional content, that content is not about that [denoting complex] itself but is rather about the entity it denotes (determines). (Levine 2004: 273)

This yields a more precise understanding of the subscript ‘M’ notation: enclosing a symbol for a complex in subscript ‘M’ indicates a non-denotative occurrence, even if that occurrence is in an entity-position. Returning to f , our function from complexes to their meanings, what we must present to it is ${}_M<\text{the } F>_M$:

- ${}_m[f]_{m/e}[{}_M<\text{the } F>_M]_e$

This yields \mathbb{M} as value, which is what we wanted. But the problem is that this explanation of the subscript ‘M’ notation is obviously circular. In explaining what

- ${}_M<\text{the } F>_M$

represents, we have appealed to

- ${}_m[f]_{m/e}[{}_M<\text{the } F>_M]_e$

which uses the very expression we wanted to explain.

Presenting the problem as one of circularity fits in very closely with what Russell writes in *OF*:

If we say “‘any man’ is a denoting complex’, ‘any man’ stands for ‘the meaning of the complex “any man”, which is a denoting concept. But this is circular; for we use “any man” in explaining “any man”. And the circle is unavoidable. For if we say “the meaning of any man”, that will stand for the meaning of the denotation of any man, which is not what we want. (*OF*: 382)

This passage uncovers two circularities, and it is important to identify which Russell is pressing. Russell’s aim in the passage is to explain the functioning of the denoting

He therefore takes it for granted that ${}_M<\text{the } F>_M$ and ${}_D<\text{the } F>_D$ are ‘distinct entities, not the same entity functioning in different ways’ (Levine 2004: 274). As I interpret (D), its *conclusion* is that ${}_M<\text{the } F>_M$ and ${}_D<\text{the } F>_D$ are not two sides of the same complex, i.e. that they are not ‘the same entity functioning in different ways’. Thus what Levine takes as a thesis to be assumed, I take as the thesis to be proved. Levine’s account (cf. especially Levine 2004: 276n.) of the role of AS* in *OF* is problematic in other respects, but we need not go into the details.

complex $\langle \text{any man} \rangle$. The proposition $\langle \langle \text{any man} \rangle \text{ is a denoting complex} \rangle$ is about—or at any rate is intended to be about—what $\langle \text{any man} \rangle$ *means* (i.e. ${}_M \langle \text{any man} \rangle_M$). In the confused terms in the quotation: “any man” stands for ‘the meaning of the complex “any man”’; in our terms: ${}_M \langle \text{any man} \rangle_M$ stands for—enables one to speak about—the meaning of the complex $\langle \text{any man} \rangle$. This is circular in that it offers an explanation (or part of an explanation) of a denoting complex in terms of a further denoting complex. Now if the problem were that we did not, in general, have an adequate grasp of denoting complexes, this would be problematic. But since that is not our problem, this kind of circularity need not concern us unduly.⁶⁰ The problematic circularity—the one to which Russell refers—concerns the fact that the denoting complex $\langle \text{any man} \rangle$ is explained in terms of *itself*: in Russell’s terms, “any man” stands for ‘the meaning of the complex “any man”’.⁶¹

Presenting the problem as one of circularity has the added advantage of affording an explanation of an otherwise very strange claim:

Thus in order to get the meaning we want, we must speak not of ‘the meaning of C ’, but of ‘the meaning of “ C ”’, which is the same as ‘ C ’ by itself. (OD: 49 D)

Is Russell suggesting that the following two claims

- ‘The meaning of “ C ”’
- ‘ C ’

are the same, or synonymous? That would make ‘the meaning of ...’ an insignificant expression when prefixed to a denoting phrase—and that is surely not the case.⁶²

⁶⁰ It would beg the question against the proponent of the theory of meaning and denotation to assume that we have no grasp of, or are not acquainted with, *any* denoting complexes.

⁶¹ Recognising and distinguishing these two respects in which the explanation is circular is important for the understanding of the GEA. It is also important in a scholarly respect, since it distinguishes my account of paragraph (D)—and of the entire GEA, in effect—from that of Makin (2000: esp. 34-7). Makin also recognises that the argument in (D) concerns symbolism, and that a circularity is involved. We disagree as to the nature of the circularity.

⁶² Levine (2004: 275) identifies an analogous circularity to that which I have identified, i.e. that ${}_M \langle \text{the } F \rangle_M$ is characterised in terms of ${}_m [f]_m / {}_e [{}_M \langle \text{the } F \rangle_M]_e$. He fails, however, to provide any explanation of how it could even be possible that an instance of the function expressed by ‘the meaning of ...’ could yield its own argument as value.

My interpretation, however, yields the result that the two expressions

- ${}_M<\text{the } F>_M$
- ${}_m[f]_{m/e}[_M<\text{the } F>_M]_e$

are equivalent: for the latter involves the complication of a non-denotative occurrence. With the distinction between denotative and non-denotative modes of occurrence in mind, we see that expressions which look as if they could not possibly be equivalent—since one is a function of the other—*can* be equivalent. The problem is that if they are equivalent, then the second expression cannot yield a non-circular explanation of the first.

The first half of paragraph (D) argues that there is no non-circular account of the subscript ‘M’ notation. The second half reinforces the point by showing that without an explanation of the subscript ‘M’ notation—and so of non-denotative occurrences—the 4E theory can give no adequate account of the role it assigns to meanings. We cannot even make sense of a meaning being denoted by another complex *without* employing the notion of a non-denotative occurrence (a notion which Russell takes himself to have shown that we do not really understand).

Russell makes the point in general terms:

‘the denotation of C ’ does not mean the denotation we want, but means something which, if it denotes at all, denotes what is denoted by the denotation we want. (*OD*: 49 D)⁶³

To render this in our terminology let Russell’s denoting complex C be $<\text{the } F>$. We have stipulated that $<\text{the } F>$ denotes \mathbb{D} . But, in asking after the denotation of $<\text{the } F>$, we are, in effect, asking after the denotation of \mathbb{D} . Why? Because the argument position in the function represented by ‘the denotation of ...’ is an entity-position. Denoting complexes standardly occur denotatively; and moreover we have been unable to provide

⁶³ Russell’s potentially confusing uses of the verb ‘to mean’ should not be taken as having a technical sense; rather they are used colloquially, as loosely equivalent to ‘to refer to’ or ‘to be about’, or something along those lines.

an explanation of non-denotative occurrence. Hence, taking f^* as a function from complexes to their denotations, ‘the denotation of $\langle \text{the } F \rangle$ ’⁶⁴ is equivalent to:

- ${}_m[f^*]_{m/e}[{}_D\langle \text{the } F \rangle_D]_e$

which is equivalent to:

- ${}_m[f^*]_{m/e}[\mathbb{D}]_e$

But this is a function of the denotation of $\langle \text{the } F \rangle$ (i.e. \mathbb{D}), rather than a function of $\langle \text{the } F \rangle$ itself, as was intended.⁶⁵ The solution, of course, is to employ the notion of a non-denotative occurrence. But the first half of (D) has argued that we cannot make sense of this notion.

Russell’s illustrative example is liable to cause confusion largely, I think, because of the way it is set out on the page. The use of indentation leads one to suppose that the complex upon which the example focuses is $\langle \text{the first line of Gray’s } \textit{Elegy} \rangle$. In fact the relevant complex is, in Russell’s terms, ‘the denoting complex occurring in the second of the above examples’, i.e. $\langle \text{the denoting complex occurring in the second example in (B)} \rangle$. The indented clause simply points out that the complex in question has, for its denotation, the complex $\langle \text{the first line of Gray’s } \textit{Elegy} \rangle$. Thus, presenting the relevant complex to the function expressed by ‘the denotation of ...’ yields (in effect):

- ${}_m[f^*]_{m/e}[{}_D\langle \text{the denoting complex occurring in the second example in (B)} \rangle_D]_e$

Which, given Russell’s indented clause, is equivalent to:

- ${}_m[f^*]_{m/e}[\langle \text{the first line of Gray’s } \textit{Elegy} \rangle]_e$

And then the problem is clear. The presentation of a complex to f^* only yields the denotation of that very complex if that complex occurs *non-denotatively*. If it occurs

⁶⁴ These are standard phrase-mentioning inverted commas.

⁶⁵ Cf. *OF*: 382-83.

denotatively the value will be the denotation of the denotation of the complex in question. Thus in Russell's example, the value of:

- $_m[f^*]_{m/e}[_D<\text{the denoting complex occurring in the second example in (B)}>_D]_e$

turns out to be a line of poetry, rather than—as was intended—<the first line of Gray's *Elegy*>.

The upshot, then, is that there is no non-circular account of the notation employed by the 4E theory. The 4E theory relies upon the notion of a non-denotative occurrence of a complex in a proposition, but we have no real understanding of such occurrences. In their absence—as shown by the second half of (D)—we really have no grip on the interaction of meanings, complexes, and denotations. On my reading, this constitutes Russell's argument in defence of (GEA 1).

4.4. Paragraph (E): retreat from the 4E to the 3E theory

Paragraph (E) sees Russell summarising the findings of paragraph (D), and offering the proponent of the theory of meaning and denotation the opportunity to retreat to the 3E theory.

The difficulty in speaking of the meaning of a denoting complex may be stated thus: The moment we put the complex in a proposition, the proposition is about the denotation; and if we make a proposition in which the subject is 'the meaning of *C*', then the subject is the meaning (if any) of the denotation, which was not intended. This leads us to say that, when we distinguish meaning and denotation, we must be dealing with the meaning; the meaning has denotation and is a complex, and there is not something other than the meaning, which can be called the complex, and be said to *have* both meaning and denotation. The right phrase, on the view in question, is that some meanings have denotations. (OD: 49-50 E)

The summary at the beginning of (E) focuses upon the difficulty in formulating a proposition about the meaning of a complex. In the terms I have employed, this amounts to the difficulty in explaining the possibility of a complex occurring in a proposition in such a way that the proposition is about the complex's meaning, rather than its denotation. In the light of this, Russell now offers his opponent an alternative

view: that the complex *just is* the meaning. On this new view—the 3E theory—a denoting phrase expresses a meaning (= denoting complex) which denotes the denotation (unless the denoting phrase is empty). Hence: ‘The right phrase [way to put it], on the view in question, is that some meanings [i.e. non-empty ones] have denotations’ (OD: 49-50 E).

4.5. Paragraphs (F) and (G): initial objection to the 3E theory

Here are paragraphs (F) and (G).

But this only makes our difficulty in speaking of meanings more evident. For suppose *C* is our complex; then we are to say that *C is* the meaning of the complex. Nevertheless, whenever *C* occurs without inverted commas, what is said is not true of the meaning, but only of the denotation, as when we say: The centre of mass of the solar system is a point. Thus to speak of *C* itself, i.e., to make a proposition about the meaning, our subject must not be *C*, but something which denotes *C*. Thus ‘*C*’, which is what we use when we want to speak of the meaning, must be not the meaning, but something which denotes the meaning. And *C* must not be a constituent of this complex (as it is of ‘the meaning of *C*’); for if *C* occurs in the complex, it will be its denotation, not its meaning, that will occur, and there is no backward road from denotations to meanings, because every object can be denoted by an infinite number of different denoting phrases. (OD: 50 F)

Thus it would seem that ‘*C*’ and *C* are different entities, such that ‘*C*’ denotes *C*; but this cannot be an explanation, because the relation of ‘*C*’ to *C* remains wholly mysterious; and where are we to find the denoting complex ‘*C*’ which is to denote *C*? Moreover, when *C* occurs in a proposition, it is not *only* the denotation that occurs (as we shall see in the next paragraph); yet, on the view in question, *C* is only the denotation, the meaning being wholly relegated to ‘*C*’. This is an inextricable tangle, and seems to prove that the whole distinction of meaning and denotation has been wrongly conceived. (OD: 50, G)

On the 4E theory, the meaning and denotation were two sides of the one complex. Having now adopted the 3E theory ‘we are to say that *C is* the meaning of the complex’ (OD: 50 F). Russell now reminds us that a ‘plain’ occurrence of the complex in a proposition entails that the proposition is about whatever the complex denotes, as in (8):

- (8) The centre of mass of the solar system is a point.

(8) expresses a proposition about p . In order to form a proposition about the meaning, then, we have to employ inverted commas, as in (9):

- (9) ‘The centre of mass of the solar system’ is a denoting complex [= meaning].

What occurs in the entity-position of (9) is not—as on the 4E theory—the same entity as occurs in entity-position in (8):

‘ C ’, which is what we use when we want to speak of the meaning, must be not the meaning, but something which denotes the meaning. (*OD*: 50 F)

Thus far, then, Russell is simply describing the 3E theory, pointing out its difference from the 4E theory.

Suppose we wish to formulate a proposition about a certain meaning (call it ‘ $M1$ ’). We need, on the 3E theory, a second meaning—call it ‘ $M2$ ’—to denote $M1$. So for instance,

- $\langle M2 \text{ is thus-and-so} \rangle$

is a proposition asserting that $M1$ is thus-and-so.

But what can we say about the relation of $M2$ to $M1$? If $M1$ were a constituent of the complex $M2$, it would occur there in an entity-position. But if $M1$ occurs as entity, it denotes its denotation. So, for instance, the complex

- ${}_m[f]_m/{}_e[M1]_e$

is a function not of $M1$, but of whatever $M1$ denotes. As Russell puts it: ‘if C [i.e. $M1$] occurs in the complex, it will be its denotation, not its meaning, that will occur’ (*OD*: 50 F). Thus if $M2$ did contain $M1$, $M2$ would amount to a function from the denotation of $M1$ back to $M1$ itself. Now if the entity denoted by $M1$ was *only* denoted by $M1$, we

would have an adequate specification of M2. But every entity is denoted by an *infinite* number of meanings and, as such, there is no ‘backward road’, in Russell’s memorable phrase, from a denotation to a *particular* meaning that denotes it.

The relation between M2 and M1—between ‘*C*’ and *C*—is now starting to look precarious. All we know about M2 is that:

- it denotes M1;
- it is not identical with M1;
- it is not expressible as a unique function of M1.⁶⁶

Russell writes:

Thus it would seem that ‘*C*’ and *C* are different entities, such that ‘*C*’ denotes *C*; but this cannot be an explanation, because the relation of ‘*C*’ to *C* remains wholly mysterious; and where are we to find the denoting complex ‘*C*’ which is to denote *C*? (OD: 50 G)

Two claims require explanation here: the ‘wholly mysterious claim’ and the ‘where are we to find ‘*C*?’’ claim.

When Russell complains that ‘the relation of ‘*C*’ to *C* remains wholly mysterious’, one might wonder whether he is not demanding too much. Going back to *PoM*, the *denoting* relation—i.e. the relation of ‘*C*’ to *C*—is taken to be fundamental, a primitive logical relation. As such, there is not, one might reasonably assume, a great deal more to be said about it: primitive logical relations are *inevitably* somewhat mysterious.

Russell would, I think, grant this point. But, in line with the general thrust of *OD*, he is going to push the question: how does the 3E theory explain the fact that (say) ‘the teacher of Plato is wise’ expresses a proposition about Socrates? Now the theory of meaning and denotation is a theory of denoting and so constitutes an account of generality—an account of those propositions that are about an entity (or entities) in

⁶⁶ I owe this way of thinking about (F) to Makin, who brings these three points out extremely clearly (Makin 2000: 25-32).

virtue of its (their) having a certain property. Hence the answer we *want* is that ‘the teacher of Plato is wise’ expresses a proposition containing <the teacher of Plato>, and that this denotes Socrates *in virtue of* his having a certain property, namely *having taught Plato*. The kind of answer we *want* will explain why, had it been not Socrates but Xanthippe who had taught Plato, ‘the teacher of Plato is wise’ would have expressed a proposition about her. What ‘remains wholly mysterious’—or so Russell is urging—is the relation between the property of *having taught Plato* as it relates to the denoting complex <the teacher of Plato> and that property as it relates to the denotation, i.e. Socrates. We might put the point as follows. Russell is not questioning the claim *that* (<the teacher of Plato> is wise) is about Socrates; he is asking *why* this is so, and is asking in expectation of answer that appeals to the fact that Socrates has a certain property. (Notice that, this being so, it is imperative for a correct interpretation of the GEA that we deny Salmon’s claim that the aim of the GEA is ‘to supplant the view that a definite description is a singular term’ (Salmon 2005: 1076).)

Russell’s ‘where are we to find ‘C’?’ claim is generally thought to raise a specifically epistemological point: that we do not know which of the infinitely many complexes ‘C’ (i.e. M2) is. This, I think, is correct. Russell is echoing the ‘no backward road’ point from (F).⁶⁷ The epistemological point is not, however, the central objection to the 3E theory. Rather, the decisive blow comes immediately afterwards, and relates to the ‘wholly mysterious’ claim:

Moreover, when *C* occurs in a proposition, it is not *only* the denotation that occurs [. . .]; yet, on the view in question [i.e. the 3E theory], *C* is only the denotation, the meaning being wholly relegated to ‘C’. (OD: 50 G)

Russell takes himself to have shown that on the 3E theory, all that the complex does is denote its denotation. There is no nuance, no twofold character, just a brute act of denotation; for the meaning has been ‘wholly relegated’ to an entirely separate entity. But, he will now claim, meaning is always relevant to a proposition—even if the proposition is (by intention) about the denotation—in a way that the 3E theory cannot account for.

⁶⁷ Salmon brings this point out very clearly (2005: 1106).

Russell's manner of expression is particularly unhelpful at this point. What he means by 'when *C* occurs in a proposition, it is not *only* the denotation that occurs' is that when a complex occurs in a proposition, it does not *just* denote its denotation, but denotes it *in virtue of* the denotation's satisfying a certain descriptive condition. This, as I have repeatedly urged, is a fundamental requirement of a theory of denoting. It is not simply a brute fact that <the teacher of Plato> denotes Socrates. It denotes him because he alone has a certain property. In entertaining a proposition containing <the teacher of Plato>, one ought to be able to recognise that it is about whoever taught Plato. Russell's claim will be that the 3E theory is unable to meet this demand. This is the 'inextricable tangle' of which he speaks. His attempt to demonstrate it is, unfortunately, no clearer than any other stage of the argument.

4.6. Paragraph (H): meaning and truth-conditional relevance

Here is paragraph (H), the final phase of the GEA:

That the meaning is relevant when a denoting phrase occurs in a proposition is formally proved by the puzzle about the author of *Waverley*. The proposition 'Scott was the author of *Waverley*' has a property not possessed by 'Scott was Scott', namely the property that George IV wished to know whether it was true. Thus the two are not identical propositions; hence the meaning of 'the author of *Waverley*' must be relevant as well as the denotation, if we adhere to the point of view to which this distinction belongs. Yet, as we have just seen, so long as we adhere to this point of view, we are compelled to hold that only the denotation can be relevant. Thus the point of view in question must be abandoned. (OD: 50-51 H)

The aim of (H) is to bring out the inextricable tangle promised in (G).

The propositions expressed by sentences (12) and (13) are clearly not identical: one has a property the other lacks, namely that George IV wished to know whether it was true.

(12) Scott was the author of *Waverley*.

⟨12⟩ ⟨Scott = <the author of *Waverley*>⟩

(13) Scott was Scott.

⟨13⟩ ⟨Scott = Scott⟩

Russell claims that this shows that ‘the meaning of “the author of *Waverley*” must be relevant as well as the denotation’ (OD: 50 H). His point is that meanings are intimately linked to *truth-conditions*.

If in ⟨12⟩ we substitute a co-denoting meaning, say <the author of *Marmion*>, we get:

(14) Scott was the author of *Marmion*.

⟨14⟩ ⟨Scott = <the author of *Marmion*>⟩

The substitution makes no difference as regards truth-value, but it makes a huge difference as regards truth-conditions. Compare Evans’ *Intuitive Criterion of Difference*:

the thought associated with one sentence *S* as its sense must be different from the thought associated with another sentence *S'* as its sense, if it is possible for someone to understand both sentences at a given time while coherently taking different attitudes to them, i.e. accepting (rejecting) one while rejecting (accepting), or being agnostic about, the other. (Evans 1982: 18-19)

In more Russellian terminology, and adapting to the case in hand: if one can understand both (12) and (14), and accept (reject) one while rejecting (accepting) or being agnostic about the other, then (12) and (14) express distinct propositions. We can demonstrate—and such a demonstration is what I take Russell to be groping (somewhat inadequately) towards in (H)—that the Intuitive Criterion of Difference is met by constructing the following example, in which substitution brings about a change in truth-value (and if the truth-value has changed the truth-condition must have changed also). So consider:

(15) George IV wished to know whether Scott was the author of *Waverley*.

(16) George IV wished to know whether Scott was the author of *Marmion*.

Suppose that George IV already knew that Scott was the author of *Marmion*. Then while (15) would express a true proposition, (16) would express a false proposition.

Russell makes this point in (H) by contrasting George IV's attitudes towards ⟨12⟩ and ⟨13⟩ rather than, as I have done, ⟨12⟩ and ⟨14⟩. His examples are apt to induce two misunderstandings. Firstly one might be led to suppose that (H) in particular, and the GEA in general, have something to do with the status of definite descriptions as compared to singular terms. But the argument has, *pace* Salmon, nothing to do with singular terms. Secondly, one might be led to suppose that the argument of (H) concerns intensional contexts. However, as I see it, intensional contexts merely provide a convenient way of demonstrating Russell's real point. That point concerns truth-conditions. ⟨12⟩ and ⟨14⟩ have different truth-conditions; and this demonstrates the respect in which meanings are 'relevant' to propositions containing denoting complexes. The complexes <the author of *Waverley*> and <the author of *Marmion*> have the same denotation but are, *qua* meanings, distinct. It is their distinctness that grounds the truth-conditional distinctness of ⟨12⟩ and ⟨14⟩.

This point may seem somewhat far removed from the text of (H); but I maintain that it *is* what Russell has in mind. This is supported by textual evidence from later work. As many commentators acknowledge, an argument that appears to be related to the GEA occurs in *KAKD*. In that discussion Russell writes as follows.

[W]hen we say 'Scott is the author of *Waverley*', the *meaning* of 'the author of *Waverley*' is relevant to our assertion. For if the denotation alone were relevant, any other phrase with the same denotation would give the same proposition. Thus 'Scott is the author of *Marmion*' would be the same proposition as 'Scott is the author of *Waverley*'. But this is plainly not the case, since from the first we learn that Scott wrote *Marmion* and from the second we learn that he wrote *Waverley*, but the first tells us nothing about *Waverley* and the second nothing about *Marmion*. Hence the meaning of 'the author of *Waverley*' as opposed to the denotation, is certainly relevant to 'Scott is the author of *Waverley*'. (*KAKD*: 216)

This is, in essentials, the same point that Russell makes in (H), or so I claim.

To reinforce the objection, it may be brought out in a slightly different way. In entertaining proposition ⟨17⟩, a subject entertains a proposition about the denotation of the denoting complex <the author of *Waverley*>.

⟨17⟩ ⟨<the author of *Waverley*> is wise⟩

But obviously the subject might entertain ⟨17⟩ without recognising that the proposition is about Scott, even though Scott *is* the denotation of the denoting complex <the author of *Waverley*>. Thus the subject entertains a thought which she is capable of recognising as being about *whoever* is the author of *Waverley*. It must also be possible for a subject entertaining ⟨17⟩ and proposition ⟨18⟩ to recognise them as distinct.

⟨18⟩ ⟨<the author of *Marmion*> is wise⟩

This ability is premised on the subject's ability to distinguish <the author of *Waverley*> and <the author of *Marmion*>. These two denoting complexes are structurally similar, but differ in terms of constituency: one has *Waverley* in the position where the other has *Marmion*. Thus the subject, in judging that ⟨17⟩ and that ⟨18⟩, must have some insight into the internal structure and constituency of the denoting complexes. The internal structure and constituency of the complexes must therefore be available to the subject in entertaining the propositions. For the subject must be able to recognise that the propositions are about whoever they are about in virtue of those person's having written *Waverley* and having written *Marmion* respectively. This is what marks out a theory of denoting out as an account of generality.

The decisive objection to the 3E theory is, then, that it renders impossible this kind of recognition. The meaning must be relevant in the sense just explained, for this is a central task of any theory of denoting. The 4E theory explains—or at least gestures towards an explanation of—this by appealing to the twofold nature of denoting complexes: complexes have two sides, one of which (i.e. the meaning) embodies the descriptive condition mentioned in the corresponding denoting phrase and satisfied by the denotation. On the 3E theory however, the complex *is* the meaning; and while its occurrence in a proposition signals that the proposition is about something else (i.e. the denotation), it does not explain a subject's ability to (say) recognise that ⟨17⟩ and ⟨18⟩ are about the same man in virtue of his satisfying two distinct descriptive conditions. The explanation we *want* is that the complex in ⟨17⟩ is such that it denotes *whoever*

wrote *Waverley* and that it, so to speak, wears this fact on its sleeve in virtue of having an additional aspect—the aspect of *meaning* (and similarly *mutatis mutandis* for the complex in ⟨18⟩). The explanation we *get*—if one may call it an ‘explanation’ at all—is simply that the complex in ⟨17⟩ denotes Scott (and similarly for ⟨18⟩). But if that is all the explanation the 3E theory offers, then one cannot say why (or even *if*) the truth-conditions of ⟨17⟩ and ⟨18⟩ differ.⁶⁸

This stage of the argument demonstrates that, on the 3E theory, the explanation of the connection between <the author of *Waverley*> and Scott amounts to no more than that the former denotes the latter. This is too brute a relation to have any real explanatory value: too brute a relation to stand alone as an account of denoting. It fails to account for the fact that one judging both that the author of *Waverley* is wise and that the author of *Marmion* is wise is able to recognise these propositions as having very different truth-conditions. Moreover the diagnosis of the problem is clear: the 3E theory goes astray in identifying the complex with the meaning. These ought to be held distinct, as on the 4E theory. This leads us to conclude that:

(GEA 2) If the meaning and denotation are *not* two sides of the one denoting complex, then we cannot explain (‘preserve’) the connection between them.

And if (GEA 2) holds, then Russell has defended both premises of his argument:

(GEA 1) If the meaning and denotation are two sides of the one denoting complex, then we cannot explain (‘preserve’) the connection between them.

(GEA 2) If the meaning and denotation are *not* two sides of the one denoting complex, then we cannot explain (‘preserve’) the connection between them.

(GEA 3) We cannot explain (‘preserve’) the connection of meaning and denotation.

⁶⁸ Thus on the 3E theory one can recognise that ⟨12⟩ and ⟨13⟩ are truth-conditionally distinct, but cannot tell whether ⟨12⟩ and ⟨14⟩ are truth-conditionally distinct. For this reason, Russell’s exposition in *KAKD* is superior to his exposition in (H).

5. The GEA versus Frege

The Central Question of *OD* is: How is it that sentences containing denoting phrases come to be about whatever it is that they are about? The conclusion of the GEA—that we cannot explain (‘preserve’) the connection of meaning and denotation—should be understood as the claim that the theory of meaning and denotation’s answer to the Central Question is inadequate.

Russell took his earlier position to be ‘very nearly the same’ (*OD*: 41n.) as that of Frege; and I have argued above that the sense in which they are in fact similar is that both appeal to a certain logical relation—denoting or determining—holding between the propositional complement of a denoting phrase and its denotation. Thus to the extent that this relation grounds Frege’s response to the Central Question, he owes a response to the GEA.

The charge, as it relates to Frege, is this. The description ‘the author of *Waverley*’ has, for its propositional complement, the *Sinn* §the author of *Waverley*§ which determines Scott. As such, the sentence ‘the author of *Waverley* is wise’ is taken by Frege to express a Thought which is about Scott in virtue of the occurrence therein of §the author of *Waverley*§. Russell’s claim in the GEA (especially in (G) and (H)) was that a 3E theory like Frege’s fails to explain the fact that this Thought is about Scott *in virtue of* his satisfying the descriptive condition *wrote Waverley*.

It certainly seems that there is a difficulty for Frege here. But it is important to recognise that differences between the overall views of Russell and Frege suggest that the theory of *Sinn* and *Bedeutung*—as opposed to Frege’s account of definite descriptions—can be maintained, if one is that way inclined.

For the pre-*OD* Russell, denoting complexes are the entities through which quantification occurs. Frege’s account of quantification is, on the other hand, independent of his theory of *Sinn* and *Bedeutung*. Now in an account of quantification, the connection between the properties which guide the quantification and the properties of the entities gathered by the quantification must be absolutely apparent. This is, in effect, the principle behind the GEA’s objection to the 3E theory in (G) and (H). Russell, who is interested in the 3E theory as, in effect, an account of quantification, is

therefore beholden to a more stringent requirement than Frege, for whom the notion of *Sinn* is *cognitive*. For Frege, the theory of *Sinn* is designed to resolve the kinds of issues arising in Frege's Puzzle (Frege 1892a): it must explain how it is possible for someone to believe that (say) Hesperus is *G* while disbelieving that Phosphorus is *G*. Thus while the theory of *Sinn* is constrained by Evans's Intuitive Criterion of Difference, it is not constrained by Russell's more stringent demands.

The danger—which the Russell of *OD* fails to avoid—is that in drawing too close a parallel between the earlier Russell and Frege, one arrives at the view that their two theories share a common purpose. Makin makes the same mistake when he claims that the two theories involve ‘essentially the same kind of theoretical device to resolve essentially the same kind of problem’ (Makin 2000: 169). We should concur rather with Levine (2004), that the two theories are differently motivated.⁶⁹ The theory of *Sinn* is intended to resolve Frege's Puzzle: the theory of meaning and denotation is intended as an account of propositions of generality. For Frege then, the theory of *Sinn* must predict that ‘the author of *Waverley* is wise’ and ‘the author of *Marmion* is wise’ express distinct Thoughts (propositions) about Scott; but it need not offer the strong account of the relations between §the author of *Waverley*§ and Scott, §the author of *Marmion*§ and Scott, that Russell demands.

Nonetheless a worry remains. It is familiar to distinguish two kinds of *Sinne*. Levine puts it this way:

we might distinguish “presentive” senses, which we grasp as a consequence of being acquainted with a certain entity in a certain way, and “descriptive” senses, which prescribe conditions which an object must fulfil in order to be determined by that sense. Whereas we cannot grasp a “presentive” sense without a prior acquaintance with the object it determines, we can grasp a “descriptive” sense without having been acquainted with the object (if any) fulfilling the condition it prescribes. (Levine 2004: 264)

A descriptive *Sinn* is like a denoting complex in that it sets up a descriptive condition that an entity must satisfy in order to be denoted (determined) by it. Insofar as Frege is

⁶⁹ Levine attributes these differences to the underlying epistemologies of Russell and Frege. While his claims regarding the epistemologies are convincing, I hold that there is a more fundamental difference in the motivations for the two theories, as discussed in the main text.

committed to an account of *Sinne* as *descriptive Sinne* he opens himself to the more stringent demand that the GEA relies upon. Grasping a proposition containing a descriptive *Sinn* would (or at least *could*) require recognising it to be about an entity picked out in virtue of its satisfying a certain descriptive condition.

Yet since Frege was committed to the view that definite descriptions are singular terms, he is not obliged to treat them as expressing descriptive *Sinne*—indeed the GEA suggests he would be well-advised not to. Presentive *Sinne* are a better option for the *Sinne* of singular terms, as they are not so much akin to embodied descriptive conditions, but are rather *ways of being acquainted with an entity*. *Sinne* thus conceived circumvent the objection to 3E theories raised in the GEA, for there is no need to appeal to a descriptive condition as something distinct from the propositional complement of the expression. The relation between the *Sinn* and the *Bedeutung* is, on this view, clear: to grasp a proposition containing the *Sinn just is* to grasp a proposition directly about the *Bedeutung* (in the only sense of ‘direct’ that a Fregean will accept as coherent⁷⁰).

The difficulty for Frege is that, to the extent that the GEA forces him to treat definite descriptions as expressing presentive *Sinne*, his conception of definite descriptions as singular terms becomes less and less plausible. If definite descriptions express presentive *Sinne*, definite descriptions will turn out to be—in Evans’s terminology—*Russellian*: their significance will depend upon their having a referent.⁷¹ But the supposition that a description’s significance depends upon its having a referent is implausible in the extreme. Thus Frege can escape the GEA, but only by giving up his claim that definite descriptions are singular terms.⁷² This is not, perhaps, so high a price to pay; and having paid it, Frege is left with an account of singular terms as expressing object-dependent *Sinne*. The GEA does not touch this view for it is, at heart, an argument about the nature of propositions of generality, not of singular terms.

⁷⁰ I assume that Fregeans deny the possibility of ‘bare’ acquaintance with entities (in obvious distinction to Russell).

⁷¹ Cf. Evans 1982: 12.

⁷² This is too quick. There are options for Frege here. He might, for instance, embrace truth-value gaps for sentences containing empty descriptions. I will not investigate the options here.

5. Toward a Solution to the Paradoxes

1. Introduction

At the end of Chapter Two, and again in passing at the beginning of Chapter Four, I gave a brief indication of the nature of Russell's attempt to use the theory of meaning and denotation to resolve the paradoxes. In this final chapter I attempt to embellish the account of the connection between the Gray's *Elegy* Argument (GEA), the theory of denoting, and the paradoxes.

In §1 I will set out certain structural similarities between Russell's thinking about classes and denoting, and draw some connections between the interplay of the notions of extension and intension in relation to the theory of classes and the theory of denoting. My comments will be somewhat speculative and are only tentatively asserted. They are intended not so much as an accurate descriptive account of Russell's thought, but as a way of making sense of its evolution.

In §2 I relate the insights garnered by Russell in *OF* and *OD* to the development of his response to the paradoxes. Here, I suggest, we see why it is important to distinguish—as I did in Chapter Three—between the theory of descriptions as presented by Russell, and the modified theory endorsed by contemporary philosophers of language. For it is the element of structurally-radical interpretive analysis—i.e. that which, I claimed, contemporary philosophers of language fail to carry over from Russell—that is the key to understanding Russell's use of the theory of descriptions (rather than the theory of meaning and denotation) in his substitutional response to the paradoxes.

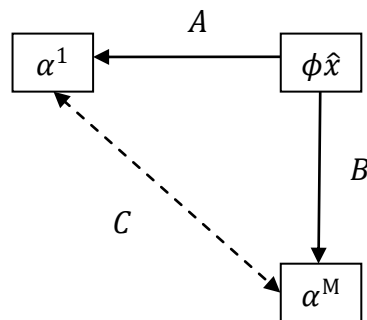
2. Classes and Denoting: Structural Similarities, Extension and Intension

I suggested in §2 of Chapter Two that Russell's thinking about classes harbours a tension which manifests itself in a complex interplay of the notions of intension and extension. Russell (*PoM*: §101, 102) had tried to state a 'mixed' paradox in terms of his distinction between classes-as-many and classes-as-one, but his attempt foundered somewhat, or so I argued, as a result of his failure to get clear as to the relation between the two kinds of class. The attempt to specify the relation between a given class-as-

many and its associated class-as-one leads inevitably to the conclusion that the mixed paradox is neither wholly extensional nor wholly intensional; for there is no specification of the relation between a class-as-many and its associated class-as-one that does not import some intensional notion or other.

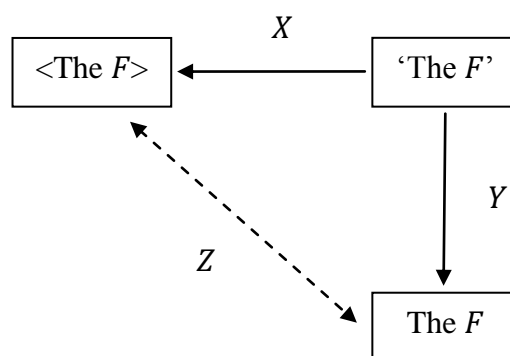
In Chapter Two I diagrammed the relations in question as follows.

Fig. 1



I offered a characterisation of the relation *C*, the direct relation holding between α^1 and α^M (i.e. a class-as-one and its related class-as-many). Similarly, I offered the following diagram of the relations involved in the theory of meaning and denotation.¹

Fig. 2



Here relation *X* is the *indication* relation, holding between the phrase $\text{'the } F \text{'}$ and its propositional complement, the denoting complex $\langle \text{the } F \rangle$. Relation *Y* is the *denoting* (in

¹ The style of diagram is owed to Demopoulos (1999).

the linguistic sense) relation between the phrase and its denotation. The GEA investigates relation Z , holding between the complex and its denotation. Superficially the similarities between Figures 1 and 2 are obvious; but we need a more detailed account of the relation of the notions of intension and extension to the theory of meaning and denotation.

It is natural to think of a denoting phrase as having an intension, namely the meaning it expresses, and an extension, namely its denotation. The GEA, however, fixes upon not the denoting phrase, but its propositional complement, the denoting complex. What is the relation between a denoting complex and its denotation, and more pertinently for present purposes, is there a purely intensional or purely extensional account of it to be had?

To hold that the relation between the complex and the denotation is purely extensional is to endorse the 3E theory. But we have seen that that theory, as an account of denoting, is insupportable. Moreover, it is insupportable precisely because it is wholly extensional, and so pays insufficient heed to the relations between the descriptive condition embodied by the complex and the satisfaction of that descriptive condition by the denotation. The upshot is that an adequate theory of the relation between the complex and its denotation must include both extensional and intensional aspects. It must take from the standpoint of extension the lesson that when the complex occurs denotatively in a proposition the proposition is about the denotation. While from the standpoint of intension must be taken the lesson that if a complex C denotes denotation D , then C embodies a descriptive condition satisfied by D . Putting these two aspects together, a correct theory of the denoting relation must hold that:

- If C denotes D , then C denotes D in virtue of the satisfaction of the descriptive condition embodied in C by D .

Accordingly, the most appropriate way of stating the relation between a complex and its denotation—between <the author of *Waverley*> and Scott, say—is not simply:

- (1) <the author of *Waverley*> denotes Scott;

but something along the following lines:

$$(2) \quad (\exists x)(Wx \ \& \ (y)(Wy \rightarrow y = x) \ \& \ (\langle \text{the author of } Waverley \rangle \text{ denotes } x))$$

That is to say: there is some unique entity satisfying the descriptive condition *wrote Waverley*, and $\langle \text{the author of } Waverley \rangle$ denotes that one. This satisfies the demand imposed from the extensional standpoint, as it tells us that $\langle \text{the author of } Waverley \rangle$ denotes a certain entity; and it also satisfies the demand imposed from the intensional standpoint, as it tells us that $\langle \text{the author of } Waverley \rangle$ denotes an entity satisfying the appropriate descriptive condition. But although (2) constitutes a response to the problems posed for the 3E theory in the GEA, it does so only by backtracking to a position in which the problems posed by the GEA for the 4E theory reappear. For (2) includes the complex $\langle \text{the author of } Waverley \rangle$; but as that complex occurs in (2) (or, better, in the proposition it expresses) it is not intended to denote its denotation, but to occur in such a way that it stands for itself.² This wants explaining, but as Russell had argued in paragraph (D) of the GEA, no satisfactory explanation is forthcoming.

The beginning of a solution to the whole problem comes in recognising that the issues raised in the GEA are to be avoided, not by striving to account for the role of any individual complex via a clause such as (2), but by providing a general method for handling any complete proposition in which such a complex occurs. This is the first step in the discovery of the theory of descriptions. Russell takes it in *OF* immediately after stumbling through the prototype GEA. The second step, the abandoning of complexes altogether, follows swiftly after. This is one of the key moments in the history of philosophy, so I quote at length.

It might be supposed that the whole matter could be simplified by introducing a relation of denoting: instead of all the complications about ‘*C*’ and *C*, we might try to put ‘*x* denotes *y*’. But we want to be able to speak of what *x* denotes, and unfortunately ‘what *x* denotes’ is a denoting complex. We might avoid this as follows: Let *C* be an unambiguously denoting complex (we may now drop the inverted commas); then we have

² That is to say: the second conjunct in (2)— $\langle \text{the author of } Waverley \rangle \text{ denotes } x$ —is about the complex $\langle \text{the author of } Waverley \rangle$ rather than Scott.

$$(\exists y) : C \text{ denotes } y : C \text{ denotes } z . \supset_z . z = y.$$

Then what is commonly expressed by $\phi' C$ will be replaced by

$$(\exists y) : C \text{ denotes } y : C \text{ denotes } z . \supset_z . z = y : \phi' y.$$

Thus e.g. ϕ' (the author of *Waverley*) becomes

$$(\exists y) : \text{'the author of } Waverley \text{' denotes } y : \text{'the author of } Waverley \text{' denotes } z . \supset_z . z = y : \phi' y.$$

Thus 'Scott is the author of *Waverley*' becomes

$$(\exists y) : \text{'the author of } Waverley \text{' denotes } y : \text{'the author of } Waverley \text{' denotes } z . \supset_z . z = y : \text{Scott} = y.$$

This, then, was what surprised people, as well it might. On this view, we shall not introduce $\iota' u$ at all, but put

$$\phi' \iota' u . = : (\exists y) : y \varepsilon u : z \varepsilon u . \supset_z . z = y : \phi' y.$$

This defines all propositions about $\iota' u$, which is all we need. (*OF*: 383-84)³

Thus Russell comes to see that he can avoid the problems concerning the vexed relation between denoting complexes and their denotations by refusing to admit denoting complexes ('we shall not introduce $\iota' u$ at all'), and instead defining the contexts in which they were previously held to occur:

On this view, 'the author of *Waverley*' has no significance at all by itself, but propositions in which it occurs have significance. Thus in regard to denoting phrases of this sort, the question of meaning and denotation ceases to exist. (*OF*: 384)

An adequate theory of the relation between a denoting complex and its denotation must include both extensional and intensional aspects. The GEA shows that the theory of meaning and denotation is unable to accommodate this. In the passage quoted

³ Dots and double-dots are used, like brackets, to indicate scope, but also for conjunction; an expression such as ' $Fz \supset_z Gz$ ' is the universal quantification ' $(z)(Fz \supset Gz)$ '; the reversed apostrophe is read 'of', hence ' $\phi' y$ ' is read ' ϕ of y '.

immediately above, Russell has, effectively, come to recognise that for phrases such as ‘the author of *Waverley*’ the problem need not arise.

OF continues with Russell extending the theory to other kinds of denoting phrases, though he fails to find a satisfactory account of ‘anything’:

The interesting and curious point is that, by driving *denoting* back and back as we have been doing, we get it all reduced to the one notion of *any*, from which I started at first.⁴ This one notion seems to be presupposed always, and to involve in itself all the difficulties on account of which we have rejected other denoting concepts. Thus we are left with the task of concocting *de novo* a tenable theory of *any*, in which denoting is not used. The interesting point which we have elicited above is that *any* is genuinely more fundamental than other denoting concepts; they can be explained by it, but not it by them. (*OF*: 387)

A concern remains, then, as to the nature of *any*, and as to the problems of meaning and denotation as they relate to the variable (Russell having taken the variable as the most general denoting concept in *PoM*):

We should, of course, simply say that “anything” is a primitive idea, if it were not for the fact that we cannot get clear as to the relation of its meaning to its denotation. (*OF*: 387-88)

As we know, shortly afterwards in *OD* Russell had come to treat the variable as fundamental. This does not, of course, constitute a ‘tenable theory of *any*, in which denoting is not used’—and this tension is never very far from the surface. G. E. Moore struck upon the issue in correspondence with Russell:

What I should chiefly like explained is this. You say ‘*all* the constituents of propositions we apprehend are entities with which we have immediate acquaintance.’ Have we, then, immediate acquaintance with the variable? And what sort of entity is it? (Moore to Russell, 23 October 1905, quoted in Hylton 1990: 256)

Russell’s response is typically modest:

⁴ It is not clear what Russell means here, but perhaps the following passage, concerning the origins of *PoM*, gives a clue: ‘I was led to a re-examination of the principles of Geometry, thence to the philosophy of continuity and infinity, and thence, with a view to discovering the meaning of the word *any*, to Symbolic Logic’ (*PoM*: xxii).

I admit that the question you raise about the variable is puzzling, as are all questions about it. The view I usually incline to is that we have immediate acquaintance with the variable, but it is not an entity. Then at other times I think it is an entity, but an indeterminate one. In the former view there is still a problem of meaning and denotation as regards the variable itself. I only profess to reduce the problem of denoting to the problem of the variable. This latter is horribly difficult, and there seem equally strong objections to all the views I have been able to think of. (Russell to Moore, 25 October 1905, quoted in Hylton 1990: 256)

The variable is a denoting concept of sorts. Thus the question arises as to the relation of its meaning to its denotation. To treat the variable as fundamental, as in *OD*, is not to answer this question but to duck it; and as the correspondence attests, Russell was quite aware of this.

With this in mind it may appear difficult to see how the theory of descriptions constitutes a decisive step away from the earlier theory. In response to this puzzle, Hylton argues that the theory of descriptions constituted progress because it allowed Russell to abandon his commitment to ‘non-propositional complexity’ (Hylton 1990: 256-263), i.e. to complex entities that neither are, nor are derived from, propositions. (The variable may remain, for it is simple rather than complex. Denoting complexes, of the kind discussed in the GEA, are eliminated.) According to Hylton, the elimination of non-propositional complexity—and especially of denoting complexes—is desirable on two counts. Firstly, denoting complexes have a certain kind of unity. This kind of unity is distinct from, but certainly no clearer than, propositional unity. Russell had no real answer to the problem of the unity of the proposition (cf. Chapter One, §2, esp. §2.4), and has no answer to the problem of the unity of non-propositional complexes either. Secondly, the inference from (to use Hylton’s example) ‘Rover is a black dog’ to ‘Rover is a dog’ is grounded in the structure of the denoting complexes <a black dog> and <a dog>. But, Hylton claims, ‘this kind of structure is not one into which we have any insight, except that it must give certain results’ (Hylton 1990: 259). So if non-propositional complexity can be eliminated in favour of propositional complexity, this is to be welcomed, for we *do* have insight into propositional structure.

One might challenge Hylton's account in various ways. In the first place, the account is misdescribed. It is not non-propositional complexity *in general* that is the problem, but simply one kind of complexity. For example, nothing in the theory of descriptions rules out realism about classes, and classes have a kind of non-propositional complexity. A more substantive criticism, however, concerns Hylton's grounding the inference from 'Rover is a black dog' to 'Rover is a dog' in the structure of denoting complexes. This might be resisted. Michael Kremer (1994: 269-272), for instance, argues that in *PoM* all denoting concepts were simple, and that inferential relations of the kind in question were grounded in the relations between denoting concepts and the class-concepts from which they are derived.

However the most telling objection to Hylton's story is that although he may have hit upon a respect in which the theory of descriptions is to be preferred to the earlier theory, there is still no explanation of the vehemence with which Russell rejects his earlier view, and no real account of the relevance of the GEA to the change in view. Russell takes the GEA to have shown the theory of meaning and denotation to involve an 'inextricable tangle' (*OD*: 50 G), that the 'whole distinction [. . .] has been wrongly conceived' (*OD*: 50 G), and that the theory 'must be abandoned' (*OD*: 51 H). It is not as if he has selected the theory of descriptions from among competing rivals: he takes the GEA to have shown that the theory of meaning and denotation is near-enough incoherent. All of this suggests that, even though it took the variable as fundamental, the theory of descriptions still somehow avoids the 'inextricable tangle' raised in the GEA. The correct account of Russell's preference for the new theory ought to reflect this fact; Hylton's, it seems to me, does not.

The correct explanation is, I think, as follows. The GEA shows that the theory of meaning and denotation has no satisfactory explanation of a certain relation: namely, that relation in virtue of which one who judges that the author of *Waverley* is wise makes a judgment about Scott *only insofar as* Scott (uniquely) satisfies the descriptive condition embodied by the denoting complex <the author of *Waverley*>. However, when discussion is restricted to variables, since the descriptive condition is, in effect, wholly absent—this being what it means to say that 'x, the variable, is essentially and wholly undetermined' (*OD*: 42)—the difficulty does not arise. It suffices that where different variables occur, one is able to distinguish them and, so to speak, keep track of

them as they occur (often multiply) in judgements and propositions. Now it may well be that Russell has no real explanation of our ability to distinguish variables one from another; but *this* is not the problem adduced in the GEA. It is the entirely unrestricted nature of the variable, the fact that it does not denote whatever it denotes in virtue of any property of that thing, that excuses it from the demands that lead to the ‘inextricable tangle’. I take it that this is the thought lying behind Russell’s claim, in his correspondence with Moore (quoted above), that he professes only ‘to reduce the problem of denoting to the problem of the variable’. My claim then, is that the GEA may be seen as bringing Russell to the realisation that the theory of meaning and denotation is unable to account for the demands imposed upon it from the standpoints of intension and extension. A denoting complex must denote its denotation in virtue of the denotation’s satisfying some descriptive condition, embodied by the complex. And it is this that the theory fails to account for.

The solution, the heart of the theory of descriptions, is to do without the denoting complex altogether, to deny that the denoting phrase has a propositional complement. That is to say, the best answer to the Central Question of *OD*—how is it that sentences containing denoting phrases come to be about whatever it is that they are about?—lies in recognising that the entire sentence should be reformulated as a complex existential claim. The kind of compositional analysis so prominent in *PoM* must be preceded by an initial stage of structurally-radical interpretive analysis (‘SR-interpretive analysis’).

This initial stage of SR-interpretive analysis will eliminate any incomplete symbols, revealing the true logical form of the proposition expressed by the sentence at hand to be a complex existential generalisation. The structure that is revealed contains only a propositional function, containing variables, and a quantifier. The only entities for which the question of meaning and denotation—the demands of the standpoints of extension and intension—could arise, namely the variables, make no descriptive demands upon their denotations. They do not denote or range over whatever they denote or range over in virtue of its satisfying some descriptive condition. Being ‘wholly undetermined’ (*OD*: 42), they range over absolutely everything whatsoever. Hence the difficulties imposed from the standpoints of intension and extension simply fall away.

We saw in Chapter Two that for Russell classes are, most fundamentally, classes-as-many, but that he also admits classes-as-one. Classes-as-one are taken as offering an account of the possibility of one class being a member of another. But as the argument of Chapter Two went, the status of the class-as-one is somewhat mysterious: it isn't a class in the primary sense (this status being reserved for classes-as-many), yet it allows one to speak about classes in the primary sense. Or again, it isn't a denoting concept, but it plays a similar role, inasmuch as its occurrence in a proposition signals that the proposition is, in some sense, about the class-as-many to which the given class-as-one stands in relation *C* (from Fig. 1). Russell's discussion of classes-as-one was deeply confused, so I claimed.

Having developed the GEA, it may⁵ have begun to slowly dawn on Russell that, just as denoted entities can be spoken of without positing strange entities occupying a precarious no-man's-land between the standpoints of extension and intension (i.e. denoting complexes), so too classes (in the primary sense) can be spoken of without recourse to entities of such dubious status as classes-as-one. Russell was not, I think, aware of the tension in his view on classes identified in Chapter Two. But that *something* was amiss must have been clear to him. Indeed in 1905 the American mathematician Maxime Bôcher had written to Russell on this subject:

The central point at issue is your 'class as one'. Your attitude towards this term is that of the realist, if I understand you correctly; mine is that of the nominalist. I cannot admit that a class is in itself an entity; it is for me *always* many entities (your 'class as many'). When we speak of it as a single entity, we are considering a new object which we associate with the class, but not the class itself. That is, the 'class as one' is merely a symbol or *name* which we may choose at pleasure. (Bôcher to Russell, 25 April 1905, quoted in Russell 1973: 130-31)

In December 1905 (so after the publication of *OD*) Russell read his paper *Difficulties* to the London Mathematical Society. He suggested there, in setting out a version of the substitutional theory, that classes-as-one be eliminated:

⁵ I put it no more strongly than this.

Instead of saying ‘The class u is a class which has only one member’, we shall say [. . .] ‘There is an entity b such that $p(x/a)$ is true when, and only when, x is identical with b ’.⁶ Here the values of x for which $p(x/a)$ is true replace the class u ; but we do not assume that these values collectively form a single entity which is the class composed of them. (*Difficulties*: 155)

Notice that this does not yet compromise the status of the class-as-many: what is eliminated is the class-as-one (‘a single entity’). The class-as-many is only officially jettisoned months later, in April 1906 in *STCR*, where Russell writes:

The theory which I wish to advocate is that classes [footnote: I use the word *class* as synonymous with *aggregate* or *manifold*.], relations, numbers, and indeed almost all the things that mathematics deals with, are ‘false abstractions’, in the sense in which ‘the present King of England’, or ‘the present King of France’ is a false abstraction. (*STCR*: 166)

(Russell’s use of ‘aggregate’ and ‘manifold’ suggests that he intends classes-as-many, as indicated by his use of these terms in that way at *PoM*: §68, 67.)

If we pursue the line of thought I am adumbrating, it will strike us as interesting that classes-as-one should be given up before classes-as-many. The uneasy status of the class-as-one, sandwiched between, on one side, the propositional function that gives rise to it, and on the other its associated class-as-many—as well as its part-extensional/part-intensional relation (as described by C in Fig. 1) to that class-as-many—makes its position analogous to that of the denoting complex. The denoting complex is sandwiched between a denoting phrase and its denotation, and its relation to that denotation incurs demands from the standpoints of extension and intension that it cannot meet. No great surprise then, if the abandoning of denoting complexes is followed by the abandoning of classes-as-one. But let me emphasise once more: I do not claim that this line of thought actually occurred to Russell; my claim is that the similarities are there to be observed and can aid our understanding of the development of Russell thought, not that they played any motivating role.

⁶ The expression $p(x/a)$ is read ‘the result of substituting x for a in p ’.

3. The GEA, the Theory of Denoting, and Russell's Progress with the Paradoxes

In the previous section I sketched some structural similarities between Russell's thinking about classes-as-one and denoting complexes. In this section I want to suggest a more concrete connection between the theory of denoting and the paradoxes.

The extension of the theory of descriptions to class-abstracts, and the possibility of dispensing with classes altogether, was clearly an important feature of Russell's eventual solution to the paradoxes. In *OF* Russell asks how, on his new theory, class abstracts such as ' $\dot{z}(\phi'z)$ ' are to be handled (*OF*: 384). According to the view of *PoM*, this class abstract can be treated as equivalent to the denoting phrase 'the class of z such that ϕz '. But the attempt to state what this denoting phrase denotes raises the following problem.⁷ What it denotes is a class-as-many; but a class-as-many, given its essentially plural nature, cannot stand as the (single) logical subject of a proposition and so, in particular, cannot be substituted for ' Y ' in a proposition of the form ' X denotes Y '. For example, if (3) is well-formed, ' α^M ' cannot genuinely be the class-as-many:

$$(3) \quad \text{'}\dot{z}(\phi'z)\text{' denotes } \alpha^M.$$

However, handling class abstracts in line with the theory of descriptions allows Russell to take:

- $u \text{ Kl } \phi$

as equivalent to:

- u is a class determined by ϕ

and then to treat the occurrence of class abstracts in propositional contexts as follows:

- $f'(\dot{z}(\phi'z)) \cdot = \cdot (\exists u) \cdot u \text{ Kl } \phi \cdot f'u \text{ Df.}$ ⁸

⁷ In addition to the problems of meaning and denotation discussed at length in Chapter Four.

⁸ See *OF*: 384.

In this way we explain formulae involving class abstracts by way of formulae in which no such expressions occur. To be sure, Russell has not by this stage arrived at a way of dispensing with classes altogether (there must be classes in some sense, else there will no value of ‘ u ’ for which ‘ u is a class determined by ϕ ’ comes out true). But it is not difficult to see him as on the way to *20.01 of *PM*:

$$*20.01. \quad f\{\hat{z}(\psi z)\} . = : (\exists \phi) : \phi!x . \equiv_x . \psi x : f\{\phi!\hat{z}\} \quad \text{Df.}$$

The Russell of *OF* has recognised that ‘class abstract talk’ can be reformulated. He has not yet managed to reformulate it in such a way that all ontological commitment to classes lapses, but he is on the way.

However, it is not the potential for the elimination of classes upon which I wish to focus. Rather, let us focus upon the relation of the insights won by the theory of denoting phrases as incomplete symbols to Russell’s attempts to develop a substitutional theory of classes in response to the paradoxes.

In *OF*, prior to discovering the theory of descriptions, Russell had used the theory of meaning and denotation to motivate the claim that in an expression such as (4):

$$(4) \quad (C)(\hat{x}) \frac{(C)(\hat{x})}{\hat{x}}$$

(i.e. ‘the result of substituting $(C)(\hat{x})$ for \hat{x} in $(C)(\hat{x})$ ’), ‘ $(C)(\hat{x})$ ’ occurs *as meaning* in its leftmost occurrence, but *as entity* in its rightmost occurrence, and hence that if (4) *does* represent the application of $(C)(\hat{x})$ to itself, this can only be ‘by accident’ (*OF*: 360). This strategy was attractive in that it opened up the possibility of a principled denial that the paradoxes are well-formed.⁹ But, as we are now in a position to recognise, (having investigated the GEA, especially paragraph (D)), the combination of the grammar of substitution with the theory of meaning and denotation was bound to fail, due to the impossibility of getting straight as to the connection of meaning and denotation.

⁹ Cf. Chapter Two, §4.2.

Russell was later to claim that his pre-*OD* attempts to implement a substitutional theory had ‘failed for want of the theory of denoting’ (Grattan-Guinness 1977: 79-80), but that subsequently, ‘as a consequence of the new theory of denoting’, he ‘found at last that substitution would work, and all went swimmingly’ (Grattan-Guinness 1977: 80).¹⁰ This development concerns the treatment of denoting phrases as incomplete symbols. An expression of the form ‘ $G(\det F)$ ’¹¹ appears to represent a proposition attributing G -ness to whatever is (are) denoted by ‘ $\det F$ ’. But the fact that ‘ $\det F$ ’ is an incomplete symbol means that ‘ $G(\det F)$ ’ in fact expresses a proposition of a radically different form to that suggested by its surface linguistic structure. No constituent of the proposition corresponds to the expression ‘ $\det F$ ’, and no constituent corresponds to ‘ $G(_)$ ’. (For recall that ‘ $G(\text{some } F)$ ’ is not the assertion that $G\hat{x}$ is true for some given value of x , but rather the assertion that $F\hat{x} \ \& \ G\hat{x}$ is true for at least one value of x .) It follows that—appearances to the contrary notwithstanding—the proposition expressed by ‘ $G(\det F)$ ’ cannot be represented as the following substitution:

$$(5) \quad G\hat{x} \frac{\det F}{\hat{x}}$$

What is wrong with (5) as a representation of ‘ $G(\det F)$ ’ is not only that it falsely represents ‘ $\det F$ ’ as having a propositional complement, but also that the proposition expressed by ‘ $G(\det F)$ ’ does not directly involve $G\hat{x}$, as does (5). (If ‘ $\det F$ ’ is ‘ $\text{some } F$ ’, ‘ $G(\det F)$ ’ directly involves not $G\hat{x}$ but $F\hat{x} \ \& \ G\hat{x}$.) As I put it in Chapter Three (§4), the predicate expression in ‘ $G(\det F)$ ’ is a *quasi-incomplete symbol*.

If Russell’s view entails that predicate expressions are quasi-incomplete symbols, the theory of descriptions is extremely ill-suited to the project of constructing a compositional semantics of natural language. But what is a weakness in the field of natural language semantics is a strength in Russell’s domain. For one of the merits of the theory of descriptions is that it yields a principled revision of the account of the logical form of a certain class of propositions. Certain substitutions at the level of symbolism will fail to be legitimate—fail to accurately represent substitutions at the propositional level—on the basis that the positions to which the substitutions are

¹⁰ I ignore the fact that the substitutional theory was ultimately unsuccessful. See Stevens 2005: ch. 3.

¹¹ Here ‘ \det ’ stands for any determiner phrase.

applied are not, at the propositional level, really there. For example, to say that the class of F s has the property G , requires that we predicate G -ness of the class of F s. But the combination of the predicate G with a denoting phrase does not yield a proposition of the form ' $G(a)$ ', with ' G ' representing the contribution of the predicate expression and ' a ' representing the contribution of the denoting phrase; rather, it yields an existential claim: that a certain complex propositional function is true for at least one value (and, to reiterate, that complex propositional function is *not* $G\hat{x}$).

The above is, albeit in different terms, the basis of Russell's substitutional approach to the paradoxes in *STCR*. In the substitutional theory, Russell does without propositional functions. Functions, he claims, are 'nothing at all without some argument' (*STCR*: 171). This, it seems to me, bears favourable comparison with the claim that predicate expressions are quasi-incomplete symbols. He continues:

Hence, we can never say, of any formula containing a variable function, that it holds 'for some value of ϕ ' or 'for all values of ϕ ', because there is no such thing as ϕ and therefore there are no values of ϕ . (*STCR*: 171)

What was previously given in terms of functions will, on the substitutional theory, be given in terms of matrices and substitutions. Thus instead of ' ϕx ' (i.e. ' x has the property ϕ '), we put ' $p/a \dot{x}$ ' (i.e. 'the result of replacing a in p by x '). (Strictly these cannot be equivalent, since the former is a sentence and the latter a definite description. To preserve equivalence, the latter should strictly be ' $p/a \dot{x}!q$ ', i.e. ' q results from p by substituting x for a in all those places where a occurs in p '. Henceforth I follow Russell's lead (*STCR*: 170n.) in ignoring this point.) The variable function ϕ is replaced by the variable entities p and a (*STCR*: 172). The two entities p and a are now held to define a class, p/a , and x will be a member of p/a if $p/a \dot{x}$ is true.¹² Now comes the key passage:

To say that x is a member of the class α is now to say that for some values of p and a , α is the matrix p/a and $p/a \dot{x}$ is true. Here, instead of the variable function ϕ , which could not be detached from its argument, we have the two variables p and a , which are entities, and may be varied. But now ' x is an x ' becomes

¹² To recapitulate the point made in parentheses above: strictly it is not $p/a \dot{x}$ that is true, but $p/a \dot{x}!q$. Note also that p/a is not a class but a matrix. Matrices do duty for classes in the substitutional theory.

meaningless, because ' x is an α ' requires that α should be of the form p/a , and thus not an entity at all. In this way membership of a class can be defined, and at the same time the contradiction is avoided. (*STCR*: 172)

This one passage draws together at least two strands of argument that Russell had previously tried, and which we have encountered in this essay.

(i) In Russell's reference to 'the variable function ϕ , which could not be detached from its argument' we catch an echo of the identification in *PoM* (§103, 104) of *quadratic forms*, and of the claim that:

the ϕ in ϕx is not a separate and distinguishable entity: it lives in the propositions of the form ϕx , and cannot survive analysis. (*PoM*: §85, 88)

This view had come with a disclaimer: 'I am highly doubtful whether such a view does not lead to a contradiction' (*PoM*: §85, 88); but it emerges in the present light as a foreshadowing of later developments. Compare:

The fundamental logical principle from which the [substitutional] theory starts is one which few people would deny. It is that, in any sentence, a single word, or a single component phrase, may often be quite devoid of meaning when separated from its context. In such a case, if the word or phrase is wrongly assumed to have an independent meaning, we get what may be called a 'false abstraction', and paradoxes and contradictions are apt to result. (*STCR*: 165)

The ϕ in ϕx does not survive analysis: it is a 'false abstraction' (it is a quasi-incomplete symbol) in much the same way as is 'the present king of France' in 'the present king of France is bald', or as is ' $\{\hat{z}(\psi z)\}$ ' in ' $f\{\hat{z}(\psi z)\}$ '.

(ii) The claim that 'we have the two variables p and a , which are entities, and may be varied' harks back to all of the problems concerning the relation between meaning- and entity-occurrence which had surfaced in the 1903-05 papers, and come to a head with the problems of meaning and denotation in the GEA. Recall Russell's claim in *OF*:

It seems likely that meaning-variation must be distinguished from entity-variation, and that two variables of which one means [i.e. occurs in meaning-

position] and the other is [i.e. occurs in entity position] can only be equal by accident, and can't be kept equal throughout variation. (*OF*: 360)

That *p* and *a* are both *entities* indicates that they are subject to precisely the same kind of variation—there is, without equivocation, only one kind of variation now. The difficulties of occurrence-as-meaning and occurrence-as-entity, and of entity- and meaning-variation, are left behind.¹³

The abovementioned points go some way, I hope, towards providing a fuller understanding of the relation between the rejection of the theory of meaning and denotation, the development of the theory of descriptions, and the consequences of developments in the theory of denoting for the attempt to solve the paradoxes.

Russell wrote of the recognition that class-symbols could be treated as incomplete symbols that it 'made it possible to see, in a general way, how a solution of the contradictions might be possible' (*MMD*: 14). Now as Graham Stevens (2005: 49) notes, it is not acceptable to simply assume that Russell is referring in the quotation to the elimination of classes: we must understand the role of the theory of descriptions in the substitutional theory. But more than this, it is not really acceptable for those us who wish to understand the development of Russell's theories of denoting in the first half of the first decade of the twentieth century to simply note the relation of the theory of descriptions to the substitutional theory. What is wanting in addition is some understanding of the reasons why the substitutional approach failed to get off the ground prior to the discovery of the theory of descriptions—in virtue of *what* is the theory of descriptions so much better suited to Russell's purposes than his earlier theory?

The answer is not—or is not *simply*—that the recognition of a class of incomplete symbols allows for the ontological elimination of classes. For as Russell pointed out

¹³ Cf. Graham Stevens: 'The most important consequence of the substitutional theory is that it disposes of the paradoxes without recourse to artificial restrictions on variation' (2005: 50).

All that is obtained by the substitutional method would still be true if there were after all such entities as classes and relations; we do not deny that there are such entities, we merely abstain from affirming that there are. (*STCR*: 188)¹⁴

Rather, the most salient point is that the theory of descriptions, and the notion of an incomplete symbol, bring with them a commitment to the idea that analysis must be, in the first instance, SR-interpretive. Entire contexts must be analysed *all at once*, rather than piecemeal, word-by-word, phrase-by-phrase. This, it will be recalled, is what marks out *Russell's* theory as distinct from its more contemporary cousin at work in the philosophy of language. It also marks out the theory of descriptions as a radically new direction in Russell's thought. For it is, most fundamentally, the commitment to the primacy of SR-interpretive analysis that distinguishes the theory of descriptions from the theory of meaning and denotation.

Wittgenstein declared that 'Russell's merit is to have shown that the apparent logical form of the proposition need not be its real form' (1922: 4.0031). A satisfactory understanding of how Russell arrived at, and subsequently implemented, this great insight demands an account of the relation of the theory of denoting to the paradoxes. This essay, I hope, goes some way toward providing just such an account.

The End.

¹⁴ Cf. *PM*: 72, and also the earlier claim in *STCR* that 'there really are no such things as classes' (*STCR*: 166).

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